Example Problem:
Proper Design of a Thermowell in Power Piping

Suggested for thermowell design suitability analysis

INTRODUCTION
Thermowells are used in measuring the temperature of a moving fluid, where the stream exerts an appreciable force and the sensitive element cannot be placed directly into the medium whose temperature is to be measured. A thermowell is a pressure tight receptacle that is adapted to receive a temperature sensing element and is provided with a variety of process connections (flanges, external threads or a machined shoulder) for tight pressure attachment to a process fitting. The purpose of this example problem is to determine whether or not a thermowell selected for temperature measurement is strong enough to withstand specific application conditions of temperature, pressure, velocity and vibration.

Standards used in the Example Problem:
ASME B31.1-2014, Power Piping
ASME PTC 19.3 TW-2010, Thermowells
ASTM A 105-14, Standard Specification for Carbon Steel Forgings for Piping Applications

THERMOWELL PARAMETERS
Proper design of a thermowell requires that the sensor mounted inside the thermowell attain thermal equilibrium with the process fluid. Thermal modeling of the sensor response is outside the scope of this Standard (refer to the latest version of PTC 19.3 for guidance). Adhering to a few general design rules will optimize the sensor performance within the constraints of the mechanical strength requirements. A high fluid-velocity rating for the thermowell requires a sufficiently high natural frequency for the thermowell and sufficiently low oscillatory stresses. Higher natural frequencies result from decreasing the unsupported length, \( L \), increasing the support-plane diameter, \( A \), and decreasing the tip diameter, \( B \). Lower oscillatory stresses result from decreasing length, \( L \), and increasing diameter, \( A \). A higher static pressure rating requires increasing the value of tip diameter \( B \). In contrast, good thermal performance favors increasing length, \( L \), and decreasing diameters, \( A \) and \( B \).

In addition, if the thermowell is to be evaluated in accordance to ASME PTC 19.3 TW, it is fit for service if it meets these four quantitative criteria:

(a) Frequency Limit. The resonance frequency of the thermowell shall be sufficiently high so that destructive oscillations are not excited by the fluid flow.  
(b) Dynamic Stress Limit. The maximum primary dynamic stress shall not exceed the allowable fatigue stress limit.  
(c) Static Stress Limit. The maximum steady-state stress on the thermowell shall not exceed the allowable stress, as determined by the Von Mises criteria.  
(d) Hydrostatic Pressure Limit. The external pressure shall not exceed the pressure ratings of the thermowell tip, shank, and flange.

In addition, the suitability of the thermowell material for the process environment (section 5) shall be considered.
PROBLEM STATEMENT

Consider a thermowell for a steam bypass line, for use under ASME B31.1, Power Piping. The designer establishes the process conditions to be:

(a) superheated steam pressure: \( P = 235 \text{ psig} \)
(b) operating temperature: \( T = 450^\circ\text{F} \)
(c) normal flow condition: \( V = 295 \text{ ft/sec} \)
(d) steam density: \( \rho = 0.499 \text{ lb/ft}^3 \)
(e) viscosity: \( \mu = 0.0171 \text{ cP} \), or using the conversion factor \( 1 \text{ cP} = 6.7197 \times 10^{-4} \text{ lb/(ft·sec)} \), \( \mu = 1.149 \times 10^{-5} \text{ lb/(ft·sec)} \)

The designer selects a thermowell with a tapered shank, and chooses to include a machined fillet at the root of the shank, which is also the support plane. For this high velocity application, the thermowell is welded directly into the process piping, with the support plane in the heat-affected zone of the weld. The nominal insertion of the thermowell into the process stream is 4 in. The unsupported length, \( L \), exceeds this nominal length due to the possible incomplete penetration of the weld (see image below).

(a) root diameter: \( A = 1.5 \text{ in.} \)
(b) tip diameter: \( B = 1.0 \text{ in.} \)
(c) fillet radius at base: \( b = 0.0 \text{ in.} \)
(d) bore: \( d = 0.26 \text{ in.} \)
(e) unsupported length: \( L = 4.06 \text{ in.} \)
(f) minimum wall thickness: \( t = 0.188 \text{ in.} \)

The designer decides to manufacture the thermowell from ASTM A 105 carbon steel to match the process piping, with the following properties:

(a) from ASME B31.1, Table C-1 (interpolated in temperature), modulus of elasticity at service temperature: \( E = 27.5 \times 10^6 \text{ psi} \)
(b) from ASME B31.1, Table A-1, maximum allowable working stress: \( S = 19,800 \text{ psi} \)
(c) thermowell construction is welded, then machined, so from ASME PTC 19.3 TW Table 6-12.3-1 (Class A, welded), fatigue endurance limit, in the high-cycle limit: \( S_f = 3,000 \text{ psi} \)
(d) from Metals Handbook Desk Edition (Davis J.R., CRC Press, 2008), mass density of carbon steel: \( \rho_m = 0.284 \text{ lb/in.}^3 \)

For the rotational stiffness of the thermowell support, \( K_M \), the designer assumes the thermowell is mounted to a thick-wall pipe (ASME PTC 19.3 TW subsection 6-6) and will use eq. (6-6-5).

For the average density of the temperature sensor, the designer chooses to use the default value found in ASME PTC 19.3 TW, \( \rho_s = 169 \text{ lb/ft}^3 \).

Reynolds and Strouhal Numbers

The Reynolds number is calculated [Eq. 6-4-3] as

\[
Re = \frac{VB\rho}{\mu} = \frac{(295 \text{ ft/sec})(1.0 \text{ in})(0.499 \text{ lb/ft}^3)}{(1.149 \times 10^{-5} \text{ lb/(ft·sec)})(12 \text{ in/ft})} = 1.068 \times 10^6
\]

For this example \( Re > 5 \times 10^5 \), and either eq. (6-4-2) or (6-4-4) gives the Strouhal number \( N_S = 0.22 \).
The force coefficients using eq. (6-4-5) are:

\[ C_D = 1.4 \]
\[ C_d = 0.1 \]
\[ C_l = 1.0 \]

**Natural Frequency Calculation.**

**Step 1.** Approximate natural frequency [eq. (6-5-1)]:

The second moment of inertia is:

\[ I = \pi (D_a^4 - d^4)/64 \]
\[ = \pi (1.25 \text{ in})^4 - (0.26 \text{ in})^4]/64 \]
\[ = 0.1196 \text{ in}^4 \]

The mass per unit length of the thermowell is:

\[ m = \rho_m \pi (D_a^2 - d^2)/4 \]
\[ = (0.284 \text{ lb/in}^3 \pi [(1.25 \text{ in})^2 - (0.26 \text{ in})^2]/4 \]
\[ = 0.3334 \text{ lb/in} \]

where \( D_a = (1.5 \text{ in} + 1.0 \text{ in})/2 = 1.25 \text{ in} \)

Calculate the approximate natural frequency of the thermowell as:

\[ f_a = \frac{1.875^2}{2\pi} \left( \frac{EI}{m} \right)^{1/2} \frac{1}{L^2} \]
\[ = \frac{1.875^2}{2\pi} \left( \frac{(27.5 \times 10^6 \text{ psi})(386.088 \text{ in} \cdot \text{lb}/(\text{lbf} \cdot \text{sec}^2)) (0.1196 \text{ in}^4)}{0.3334 \text{ lb/in}} \right)^{1/2} \frac{1}{(4.06 \text{ in})^2} \]
\[ = 2095 \text{ Hz} \]

where

\( E \) = the elastic modulus at the operating temperature
\( I = \pi (D_a^4 - d^4)/64 \), which is the second moment of inertia
\( L \) = unsupported length of the thermowell
\( m = \rho_m \pi (D_a^2 - d^2)/4 \) is the mass per unit length of the thermowell

The conversion factor 386.088 in.-lb = 1 lbf·sec² is necessary when \( E \) is given in units of pounds per square inch (equivalent to lbf/in²). (See para. 6-5.3, Step 2 and the Nonmandatory Appendix A.)

**Step 2:** Use the correlations of subsection 6-5 to correct for deviations from the approximate slender-beam theory:

\[ H_f = \frac{0.99 \left[ 1 + (1 - B/A) + (1 - B/A)^2 \right]}{1 + 1.1(D_a/L)^{3(1-0.8(d/D_a))}} \]
\[ = \frac{0.99 \left[ 1 + (1 - 0.6667) + (1 - 0.6667)^2 \right]}{1 + 1.1(0.3079)^{3(1-0.8(0.2080))}} \]
\[ = 1.352 \]

where

\( B/A = (1.0 \text{ in})/(1.5 \text{ in}) = 0.6667 \)
\( D_a/L = (1.25 \text{ in})/(4.06 \text{ in}) = 0.3079 \)
\( d/D_a = (0.26 \text{ in})/(1.25 \text{ in}) = 0.2080 \)
**Step 3:** Correct for the fluid mass:

\[
H_{a,f} = 1 - \frac{\rho}{2\rho_m} = 1 - \frac{\left(0.499 \text{ lb/ft}^3\right)}{2\left(0.284 \text{ lb/in}^3\right)\left(1728 \text{ in}^3/\text{ft}^3\right)} = 0.9995
\]

**Step 4:** Correct for the sensor mass:

\[
H_{a,s} = 1 - \frac{\rho_s}{2\rho_m} = 1 - \frac{\left(169 \text{ lb/ft}^3\right)}{2\left(0.284 \text{ lb/in}^3\right)\left(1728 \text{ in}^3/\text{ft}^3\right)\left(4.808^2 - 1\right)} = 0.9922
\]

where

\[
D_j/d = (1.25 \text{ in.)}/(0.26 \text{ in.)}) = 4.808
\]

**Step 5:** The lowest-order natural frequency of the thermowell with ideal support [eq. (6-5-6)] is given by:

\[
f_n = H_f H_{a,f} H_{a,s} f_s = (1.352)(0.9995)(0.9922)(2095 \text{ Hz}) = 2809 \text{ Hz}.
\]

**Step 6:** Correct for foundation compliance [eq. (6-6-5)]:

\[
H_c = 1 - (0.61) \frac{(A/L)}{\left(1 + 1.5(b/A)\right)^2} = 1 - (0.61) \frac{(0.3695)}{\left(1 + 1.5(0)\right)^2} = 0.7746
\]

where

\[
A/L = (1.5 \text{ in.)}/(4.06 \text{ in.)}) = 0.3695
\]

\[
b/A = (0.0 \text{ in.)}/(1.5 \text{ in.)}) = 0.0
\]

The in situ natural frequency of the mounted thermowell [eq. (6-6-1)] is given as

\[
f_n^c = H_c f_n = (0.7746)(2809 \text{ Hz}) = 2176 \text{ Hz}
\]
Scrubton Number Calculation
Because the Reynolds number exceeds $10^5$, the general frequency limits of para. 6-8.3 apply and no calculation of Scrubton number is needed. The calculation is included here as an example. We take a conservative value of 0.0005 for the damping factor, $\zeta$, used in eq. (6-8-1):

$$N_{Sc} = \pi^2 \zeta \left( \frac{\rho_m}{\rho} \right) \left( 1 - \left( \frac{d}{B} \right)^2 \right)$$

$$= \pi^2 (0.0005) \left( \frac{0.284 \text{ lb/in}^3}{(0.499 \text{ lb/ft}^3)(5.787 \times 10^{-4} \text{ ft}^3/\text{in}^3)} \right) \left( 1 - 0.2600^2 \right)$$

$$= 4.525.$$  

where

d/B = (0.26 \text{ in})/(1.0 \text{ in}) = 0.26

Although $N_{Sc}$ is greater than 2.5, the Reynolds number exceeds $10^5$, and the in-line resonance cannot be assumed to be suppressed.

Frequency Limit Calculation.

Step 1: From eq. (6-4-1), the vortex shedding rate with a Strouhal number $N_S = 0.22$ and at the normal flow condition is

$$f_s = \frac{N_S V}{B}$$

$$= (0.22)(295 \text{ ft/sec})(12 \text{ in/ft})$$

$$= 778.8 \text{ Hz}$$

Step 2: Check that the natural frequency of the mounted thermowell is sufficiently high. In the present example, the thermowell passes the most stringent frequency limit [eq. (6-8-7)]:

$$f_s < 0.4f_n^c ,$$

$$778.8 \text{ Hz} < 870.2 \text{ Hz} = 0.4(2176 \text{ Hz})$$

In this case, no calculation of cyclic stress at in-line resonance is needed, because the forced or Strouhal frequency is less than the in-line resonance frequency. However, for the sake of completeness, calculation of this quantity is included in para. 8-1.5.

Cyclic Stress at the In-line Resonance.

Step 1: Use eqs. (6-8-3) and (6-8-4) to establish the flow velocity corresponding to the in-line resonance:

$$V_{IR} = \frac{Bf_n^c}{2N_S}$$

$$= \frac{(1.0 \text{ in})(12 \text{ in/ft}^{-1})(2176 \text{ Hz})}{2(0.22)}$$

$$= 412.0 \text{ ft/sec}$$
Step 2: Evaluate cyclic drag stress at the root. The magnification factor \( F'_{M} \) for the drag/in-line resonance is set at 1000 (see paras. 6-8.3, Step 1; and 6-9.2). Begin by evaluating the value of \( G_{SP} \), using eq. (6-10-7):

\[
G_{SP} = \frac{16L^2}{3\pi A^2 \left[ 1 - (d/A)^2 \right]} \left[ 1 + 2(B/A) \right]
= \frac{16(4.06 \text{ in})^2}{3\pi(1.5 \text{ in})^2 \left( 1 - 0.1733^2 \right)} \left[ 1 + 2(0.6667) \right]
= 29.05
\]

where
\[d/A = (0.26 \text{ in})/(1.5 \text{ in}) = 0.1733\]

From eq. (6-3-3), the force per unit area due to cyclic drag is

\[
P_d = \frac{1}{2} \rho C_d V_{IR}^2
= \frac{1}{2} \left( \frac{0.499 \text{ lb/ft}^3 \times 10^{-4} \text{ ft}^3/\text{in}^3 \times 0.1}{386.088 \text{ in} \cdot \text{lb}/(\text{lbf} \cdot \text{sec}^2)} \right) \left[ (412.0 \text{ ft/sec})(12 \text{ in/ft}) \right]^2
= 0.9143 \text{ psi}
\]

where the conversion factor 386.088 in.-lb = 1 lbf·sec² is included to give a final answer in units of pounds per square inch (psi).

The cyclic stresses due to cyclic drag [eq. (6-10-6)] at the in-line resonance condition are:

\[
S_d = G_{SP} F'_M P_d
= 29.05(1000)(0.9143 \text{ psi})
= 26,560 \text{ psi}
\]

Step 3: Evaluate the stress concentration factor from eq. (6-12-4):

\[K_t = 2.2\]

Step 4: Evaluate combined drag and lift stresses, with lift stress set to zero, [eq. (6-12-3)]:

\[
S_{o,\text{max}} = K_t \left( S_d^2 + S_L^2 \right)^{1/2} = K_t S_d
= 58,430 \text{ psi}
\]

Step 5: Evaluate the temperature de-rating factor from eq. (6-12-6):

\[
F_T = E(T)/E_{\text{ref}}
= \frac{27.5 \times 10^6 \text{ psi}}{29.3 \times 10^6 \text{ psi}}
= 0.9386
\]

The environmental de-rating factor \( F_E \) is taken as unity for steam service.
Step 6: Compare the predicted stress with the fatigue stress limit, given by the right hand side of eq. (6-12-5):

\[
F_T F_E S_f = (0.9386)(1.0)(3000 \text{ psi})
\]

\[
= 2816 \text{ psi}
\]

The fatigue stress limit, 2816 psi, is less than the combined stress, 58,430 psi. The thermowell would not pass the cyclic stress condition for steady state operation at the in-line resonance, corresponding to a fluid velocity of 412.0 ft/sec, if the vortex shedding frequency \( f_s \) had been greater than \( 0.4 f_n' \) (see para. 8-1.4, Step 2).

**Steady-state Stress at the Design Velocity**

**Step 1:** Evaluate the radial, tangential, and axial stresses due to the external pressure, at the location of maximum stress [eqs. (6-11-1 through (6-11.3)):

\[
S_r = P = 235 \text{ psi}
\]

\[
S_t = P \frac{1+(d/A)^2}{1-(d/A)^2} = (235 \text{ psi}) \frac{1+(0.1733)^2}{1-(0.1733)^2}
\]

\[
= 249.6 \text{ psi}
\]

\[
S_a = \frac{P}{1-(d/A)^2} = (235 \text{ psi}) \frac{1}{1-(0.1733)^2}
\]

\[
= 242.3 \text{ psi}
\]

**Step 2:** Evaluate steady-state drag stress at the root. First, evaluate the steady-state drag force per unit area:

\[
P_D = \frac{1}{2} \rho C_D V^2
\]

\[
= \frac{1}{2} \left( \frac{0.499 \text{ lb/ft}^3}{(386.088 \text{ in.}-\text{lb}/(\text{lbf} \cdot \text{sec}^2))} \right) \left( 5.787 \times 10^{-4} \text{ ft}^3/\text{in.}^3 \right) \left( 1.4 \right) \left( 295 \text{ ft/sec} \right) \left( 12 \text{ in./ft} \right)^2
\]

\[
= 6.561 \text{ psi}
\]

where the conversion factor 386.088 in.-lb = 1 lbf·sec² is included to give a final answer in units of pounds per square inch (psi).

**Step 3:** Evaluate the steady-state stress due to the drag force [eq. (6-10-4)):

\[
S_D = G_{SP} P_D
\]

\[
= 29.05(6.561 \text{ psi})
\]

\[
= 190.6 \text{ psi}
\]

**Step 4:** Before using the Von Mises criterion to assess the stress limit at the root, compute the maximum stress given by eq. (6-12-1):

\[
S_{\text{max}} = S_D + S_a
\]

\[
= 432.9 \text{ psi}
\]
Step 5: Compute the left hand side of the Von Mises criteria, [eq. (6-12-2)]:

\[
\text{LHS} = \sqrt{\frac{(S_{\text{max}} - S_{L})^2 + (S_{\text{max}} - S_{T})^2 + (S_{L} - S_{T})^2}{2}}
\]

\[
= 191.0 \text{ psi}
\]

Step 6: Compute the stress limit given by the right hand side (RHS) of the Von Mises criteria [eq. (6-12-2)]:

\[
\text{RHS} = 1.5S
\]

\[
= 1.5(19,800 \text{ psi})
\]

\[
= 29,700 \text{ psi}
\]

The Von Mises stress, 191.0 psi, does not exceed the stress limit, 29,700 psi, and the thermowell passes the steady-state stress criterion.

Dynamic Stress at the Design Velocity

Step 1: The magnification factor for the lift (transverse) and drag (in-line) resonances are given by eqs. (6-9-1) and (6-9-2), respectively:

\[
r = \frac{f_s}{f_n} = \frac{778.8 \text{ Hz}}{2176 \text{ Hz}} = 0.3580
\]

\[
F_M = \frac{1}{1 - r^2} = \frac{1}{1 - 0.3580^2} = 1.147
\]

\[
r' = \frac{2f_s}{f_n} = \frac{2(778.8 \text{ Hz})}{2176 \text{ Hz}} = 0.7159
\]

\[
F_M' = \frac{1}{1 - (0.7159)^2} = 2.052
\]

Step 2: Evaluate the dynamic drag and lift stresses at the root. Using eq. (6-3-3), the force per unit area due to cyclic drag and lift are:

\[
P_d = \frac{1}{2} \rho C_d V^2
\]

\[
= \frac{1}{2} \frac{0.499 \text{ lb/ft}^3 \left(5.787 \times 10^{-4} \text{ ft}^3/\text{in}^3\right)(0.1)}{(386.088 \text{ in} \cdot \text{lb/}(\text{lbf} \cdot \text{sec}^2)) \left[(295 \text{ ft/sec})(12 \text{ in/ft})\right]^2}
\]

\[
= 0.4686 \text{ psi}
\]

\[
P_l = \frac{1}{2} \rho C_l V^2
\]

\[
= \frac{1}{2} \frac{0.499 \text{ lb/ft}^3 \left(5.787 \times 10^{-4} \text{ ft}^3/\text{in}^3\right)(1.0)}{(386.088 \text{ in} \cdot \text{lb/}(\text{lbf} \cdot \text{sec}^2)) \left[(295 \text{ ft/sec})(12 \text{ in/ft})\right]^2}
\]

\[
= 4.686 \text{ psi}
\]
The cyclic stresses due to drag and lift [eqs. (6-10-5) and (6-10-6)] are:

\[ S_d = G_{SP} F_M P_d \]
\[ = (29.05)(0.4686 \text{ psi}) \]
\[ = 27.93 \text{ psi} \]

\[ S_l = G_{SP} F_M P_l \]
\[ = (29.05)(4.686 \text{ psi}) \]
\[ = 156.1 \text{ psi} \]

The concentration factor is identical to the value calculate in para. 8-1.5, Step 3, \( K_t = 2.2 \).

**Step 3**: Evaluate combined drag and lift stresses, [eq. (6-12-3)]:

\[ S_{o,\text{max}} = K_t \left( S_d^2 + S_l^2 \right)^{1/2} \]
\[ = 2.2 \left( (27.93 \text{ psi})^2 + (156.1 \text{ psi})^2 \right)^{1/2} \]
\[ = 348.9 \text{ psi} \]

**Step 4**: The temperature de-rating factor is identical to the value calculated in para. 8-1.5, Step 5, \( F_T = 0.9386 \). The environmental de-rating factor \( F_E \) is taken as unity for steam service.

**Step 5**: Compare the predicted stress with the fatigue stress limit, given by the right hand side of eq. (6-12-5):

\[ F_T F_E S_f = (0.9386)(1.0)(3000 \text{ psi}) \]
\[ = 2816 \text{ psi} \]

The predicted stress of 348.9 psi is below the fatigue stress limit, and the thermowell passes the dynamic stress criterion.

**Pressure Stress**

**Step 1**: Compute the external pressure rating for the shank using eq. (6-13-1):

\[ P_c = 0.66 S \left[ \frac{2.167}{2B/(B - d)} - 0.0833 \right] \]
\[ = 0.66(19,800 \text{ psi}) \left[ \frac{2.167}{2(1.0 \text{ in})/(1.0 \text{ in} - 0.26 \text{ in})} - 0.0833 \right] \]
\[ = 9389 \text{ psi} \]

**Step 2**: Compute the external pressure rating for the tip using eq. (6-13-2):

\[ P_t = \frac{S}{0.13} \left( \frac{t}{d} \right)^2 \]
\[ = 19,800 \text{ psi} \left( \frac{0.188 \text{ in}}{0.26 \text{ in}} \right)^2 \]
\[ = 79,630 \text{ psi} \]
The pressure rating for the thermowell is the lesser of $P_t$ and $P_e$, which is 9389 psi in the present case. This rating exceeds the 235 psi operating pressure, and the thermowell passes the external pressure criterion.

The designer was pleased to see that for this set of process conditions, the thermowell was found to be fit for service.