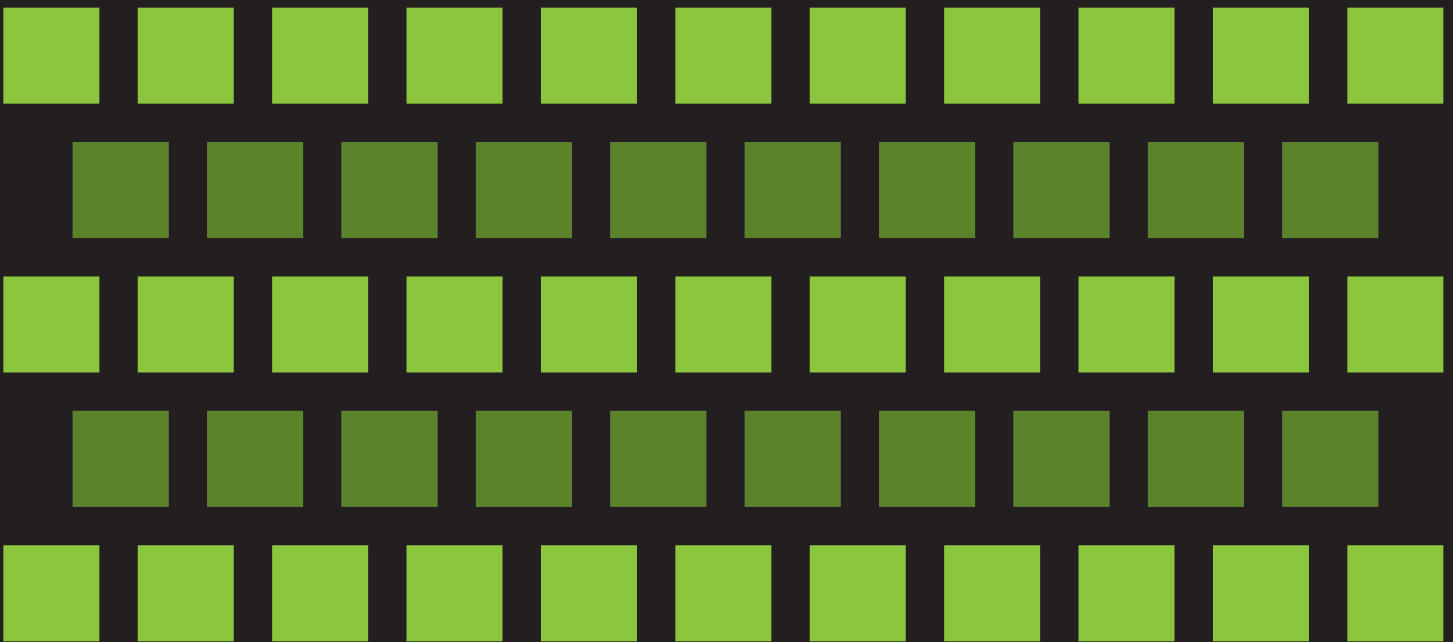


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# COMPARISON AND VALIDATION OF CREEP-BUCKLING ANALYSIS METHODS



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## FOREWORD

This document was developed under a research and development project which resulted from ASME Pressure Technology Codes & Standards (PTCS) committee requests to identify, prioritize, and address technology gaps in current or new PTCS Codes, Standards and Guidelines. This project is one of several included for ASME fiscal year 2008 sponsorship which are intended to establish and maintain the technical relevance of ASME codes & standards products. The specific project related to this document is project 07-11 (BPVC#5), entitled “Comparison and Validation of Creep-Buckling Analysis Methods.”

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## ABSTRACT

This report provides comparisons of creep-buckling calculations and provides guidance on approximate methods which are feasible for design. This report includes a discussion of the various creep models, presents creep buckling analysis techniques, and provides several comparative example calculations.

The techniques discussed in this report include:

1. Baseline analysis. Finite element creep analysis with different creep models and full non-linear strain-displacement (geometrical) analysis.
2. Critical strain technique. Elastic buckling strain defines the creep buckling strain.
3. Tangent/secant modulus approaches. Combinations of tangent and secant moduli of the isochronous stress-strain curve are used in calculations that reduce to elastic buckling calculations in the elastic case.
4. Use of an isochronous stress-strain curve in a limit/instability analysis of the imperfect structure. An instability (buckling) analysis would be in principle the same as Technique 3, and should generate the same answer. Adding plastic collapse as a failure mode ensures that the yield strength of the structure is not exceeded. This analysis therefore reflects the failure modes which are covered by the baseline technique.