

TABLE OF CONTENTS

Preface	ix
Overview	xi
The Method of Weighted Residuals (MWR)	xi
MWR Example Problem: FDM, FVM, FEM, BEM and MM	xiv
Finite Difference Method (FDM) – Collocation MWR with Local Polynomial Trial Functions	xv
Finite Volume Method – Subdomain MWR with Local Polynomial Trial Functions	xviii
Finite Element Method – Galerkin MWR with Local Polynomial Trial Functions	xxi
Boundary Element Method – Collocation MWR of Boundary Integral Equation	xxv
Meshless Method – Collocation MWR with Global Radial-Basis Function (RBF) Trial Functions	xxviii
References	xxxiii
Appendix A	xxxii
Derivation of the 1D Fundamental Solution for $T'' + T = -\delta(x - x_i)$	xxxii
Appendix B-MATLAB	xxxv
Appendix C-MAPLE	xlix

PART I THE FINITE ELEMENT METHOD	1
Chapter 1 Introduction	3
Chapter 2 Governing Equations	5
2.1 Mass Conservation	5
2.2 Navier-Stokes	5
2.3 Energy Conservation	5
2.4 Mass Transport	6
2.5 Boundary Conditions	6
Chapter 3 The Finite Element Method	7
3.1 Error in Finite Element Approximation	8
3.2 One-Dimensional Elements	8
3.2.1 Linear Element	8
3.2.2 Quadratic and Higher Order Elements	9

3.3	Two-Dimensional Elements	10
3.3.1	Triangular Elements	10
3.3.2	Quadrilateral Elements	12
3.3.3	Isoparametric Elements	13
3.4	Three-Dimensional Elements	17
3.5	Quadrature	18
3.6	Reduced Integration	20
3.7	Time Dependence	21
3.7.1	The θ Method	21
3.7.2	Mass Lumping	22
3.8	Petrov-Galerkin Method	23
3.9	Taylor-Galerkin Method	25
Chapter 4	Mesh Generation	27
4.1	Mesh Generation Guidelines	27
4.2	Bandwidth	29
4.3	Adaptation	30
4.3.1	Mesh Regeneration	31
4.3.2	Element Subdivision	32
4.3.3	Adaptation Rules	33
4.3.4	Mesh Adaptation Example	34
Chapter 5	Fluid Flow Applications	37
5.1	Constant-Density Flows	38
5.1.1	Mixed Formulation	38
5.1.2	Fractional Step Method	42
5.1.3	Penalty Function Formulation	43
5.1.4	Calculation of Pressure	44
5.1.5	Open Boundaries	44
5.2	Free Surface Flows	45
5.3	Flows in Rotating Systems	46
5.4	Isothermal Flow Past a Circular Cylinder	47
5.5	Turbulent Flow	48
5.5.1	Large Eddy Simulation (LES)	51
5.5.2	Subgrid-Scale (SGS) Modeling	54
5.6	Compressible Flow	55
5.6.1	Supersonic Flow Impinging on a Cylinder	57
5.6.2	Transonic Flow Through a Rectangular Nozzle	58
Chapter 6	List of Commercial Codes	61
Chapter 7	Conclusion	65
	References	66
	APPENDIX A	71
	Symbols	71
	Subscripts	73
	Superscripts	73
	APPENDIX B	75
B.1	Matrix Equations and Solution Method	76
B.2	Temporal Evolution of the Semi-Implicit Scheme	76
B.2.1	Momentum	76
B.2.2	Continuity	77
B.2.3	Energy	78

B.2.4	Turbulent Kinetic Energy and Specific Dissipation Rate ($\kappa-\omega$)	78
B.2.5	Matrix Formulation	79
	References	80
<hr/>		
PART II THE BOUNDARY ELEMENT METHOD		81
Chapter 1 Introduction		83
Chapter 2 BEM Fundamentals		85
2.1	A Familiar Example: Green's Third Identity for Potential Problems	85
2.2	The 2D Heat Conduction Problem	87
2.3	Generating the Integral Equation: Weighting Function and Green's Second Identity	88
2.4	Analytical Solution: Green's Function Method and the Auxiliary Problem	90
2.5	Numerical Solution: The BEM and the Boundary Integral Equation	93
	Appendix A Derivation of the Green's Function for the 2D Problem in a Square	106
	Appendix B Derivation of the Green's Free Space (Fundamental) Solution to the Laplace Equation	107
Chapter 3 Numerical Implementation of the BEM		109
3.1	Two-Dimensional Boundary Elements	109
3.2	Three-Dimensional Boundary Elements	115
3.3	Adaptive Quadrature in 3D	119
3.4	Numerical Solution of the BEM Equations	121
	Appendix A Conjugate Gradient and GMRES MATHCAD Pseudo-Codes	123
Chapter 4 Steady Heat Conduction with Variable Heat Conductivity		129
4.1	Nonlinear Thermal Conductivity	129
4.2	Anisotropic Heat Conductivity	131
4.3	Non-Homogenous Thermal Conductivity	133
Chapter 5 Heat Conduction in Media with Energy Generation		139
5.1	Special Form of Generation Leading to Contour Integrals	139
5.2	Use of Particular Solutions	141
5.3	The Dual Reciprocity Boundary Element Method	142
Chapter 6 Applications of the BEM to Heat Transfer and Inverse Problems		149
6.1	Axi-Symmetric Problems	149
6.2	Heat Conduction in Thin Plates and Extended Surfaces	151
6.3	Conjugate Heat Transfer	154
6.4	Large-Scale Heat Transfer	157
6.5	Non-Homogeneous Heat Conduction: Generalized BIE	162
6.6	Inverse Problems Applications of the BEM	166
Chapter 7 Conclusion		173
	References	173

PART III THE MESHLESS METHOD	179
Chapter 1 Introduction and Background	181
Chapter 2 Radial-Basis Function (RBF) Interpolation	183
Chapter 3 The Localized Collocation Meshless Method (LCMM) Framework	187
Chapter 4 The Moving Least-Squares (MLS) Smoothing Scheme	193
Chapter 5 The Finite-Differencing Enhanced LCMM	195
Chapter 6 Upwinding Schemes	199
6.1 One-Dimensional LCMM Upwinding Test	200
6.2 Two-Dimensional LCMM Upwinding Test for an Inclined Wave	203
6.3 Two-Dimensional LCMM Upwinding Test for a Turning Wave	205
Chapter 7 Automatic Point Distribution	207
Chapter 8 Parallelization	209
Chapter 9 Applications	211
9.1 Incompressible Fluid Flow and Conjugate Heat Transfer	211
9.1.1 Decaying Vortex Flow	215
9.1.2 Lid-Driven Flow in a Square Cavity	218
9.1.3 Air Jet into a Square Cavity	220
9.1.4 Conjugate Heat Transfer between Parallel Plates	221
9.1.5 Conjugate Heat Transfer Flow over a Rectangular Obstruction	223
9.1.6 Conjugate Film-Cooling Heat Transfer	225
9.1.7 Flow over a Cylinder	227
9.1.8 Steady Blood Flow through a Femoral Bypass	229
9.1.9 Pulsatile Blood Flow through a Femoral Bypass	233
9.2 Natural Convection	235
9.2.1 Buoyancy-Driven Flow in a Square Cavity	236
9.2.2 Buoyancy-Driven Flow of Liquid Aluminum in a Rectangular Cavity	238
9.3 Turbulent Fluid Flows	239
9.3.1 Turbulent Flow over a Flat Plate	241
9.3.2 Turbulent Flow over a Backward-Facing Step	242
9.4 Compressible Fluid Flows	243
9.4.1 Subsonic and Supersonic Smooth Expanding Diffuser	245
9.4.2 Characteristic Nozzle Flow	247
9.4.3 Subsonic and Supersonic Flow Past an Airfoil	248
9.4.4 Turbulent Wake Flow	251
9.5 Two-Phase Flow	252
9.5.1 Dam-Breaking Test of Two-Phase Flow Formulation	253
9.6 Solid Mechanics and Thermo-Elasticity	254
9.6.1 Cantilever Beam under Constant Distributed Load	256
9.6.2 Cortical Bone with Fixation Element under Bending Moment	256
9.7 Porous Media Flow and Poro-Elasticity	258
9.7.1 Rectangular Poro-Elastic Medium	260
9.7.2 Air Flow Coupled with Poro-Elastic Balloon	260
9.7.3 Coupled Tracheo-Bronchial Poro-Elastic Lung	262
9.7.4 Groundwater Flow through a Poro-Elastic Levee	263
Chapter 10 Conclusions	265
References	266