

Introduction to VVUQ | Example Projectile Motion

Task Group on VVUQ Concepts in Engineering Education

ASME Codes & Standards | Committee on Verification, Validation, and Uncertainty Quantification

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Outline

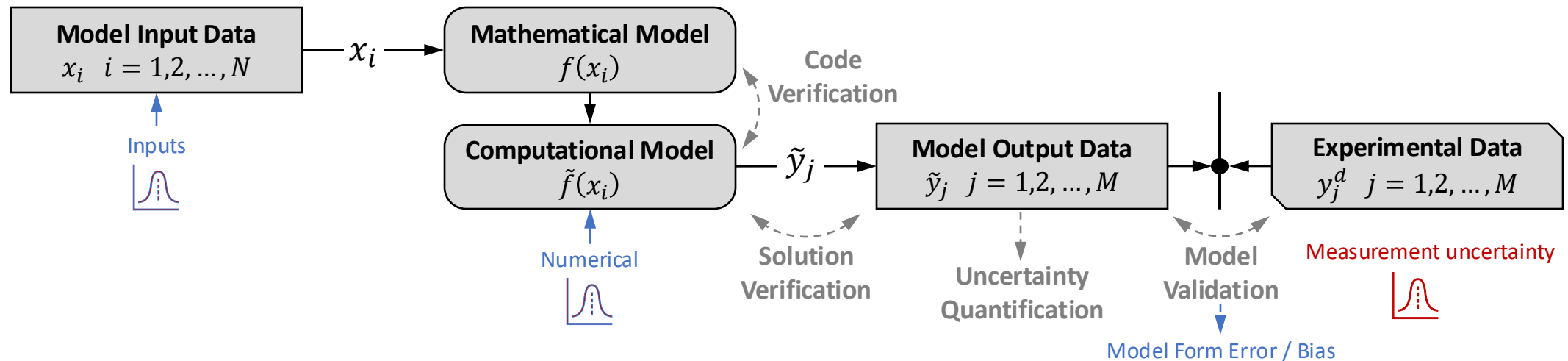
- The VVUQ Process
- Problem & Model Definition
- Sources of Uncertainty
- Solution Verification
- Uncertainty Quantification
- Validation Assessment
- Decision-Making
- Conclusions

Notes:

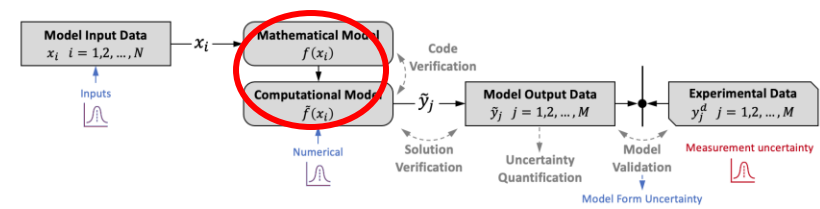
- This example is based on the VVUQ Process outlined in the companion slides found online [1]
- A paper related to this example is available [2]

The VVUQ Process

- The VVUQ Process [1-6] involves estimation of error and uncertainty sources from the model and measurements and subsequent propagation of uncertainty through the model to its predicted outputs, or Quantities of Interest (QoI).
- The sources of uncertainty from the **model** and **measurements** include:
 - **Numerical (σ_{num})**: estimated during *solution verification*, e.g., mesh refinement studies
 - **Inputs (σ_{in})**: impact of model inputs on outputs, estimated via *UQ*, e.g., Monte Carlo sampling
 - **Model Form / Bias (δ)**: error in the correctness of the model definition, estimated during model validation
 - **Measurements (σ_d)**: measurement system uncertainty



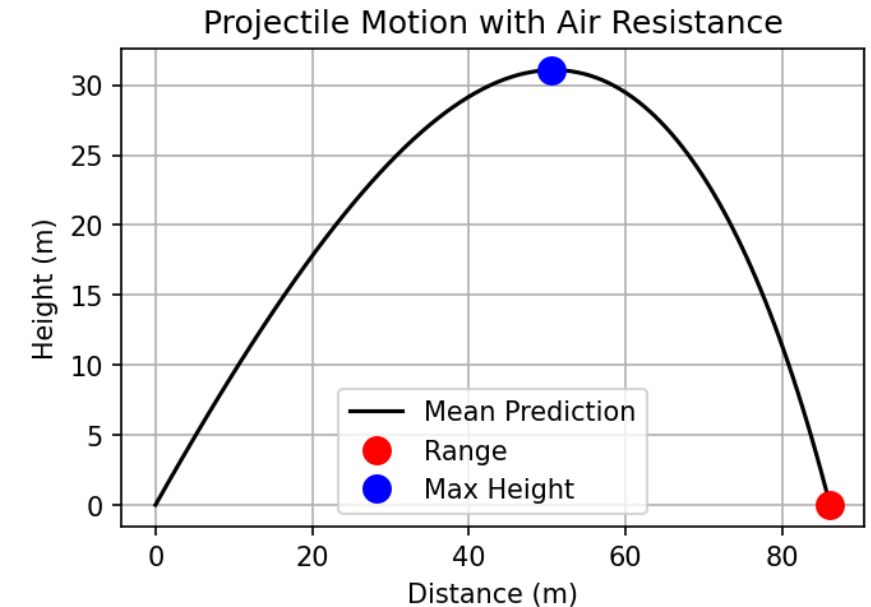
Problem & Model Definition



- **Problem:** Develop a model to predict a spherical projectile’s flight path and then carry out VVUQ to estimate how accurately the model can predict the projectile’s **range** [7-9].
- **Model Definition:** The model is defined in two steps
 - The *mathematical model* is 2D and further assumes
 - the object is a point mass, and the ground is assumed to be planar
 - 2 forces: quadratic drag & gravity (no wind, no spin)
 - model inputs (table below right) include 4 stochastic variables

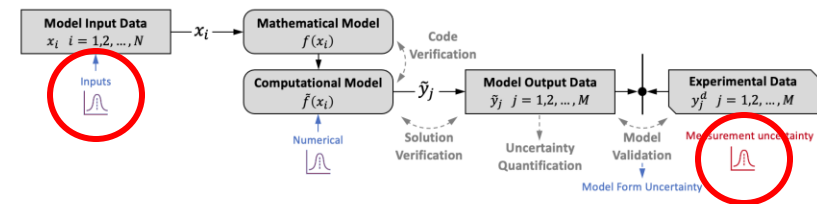
Forces	Force of gravity: $\mathbf{F}_g = -mg\hat{\mathbf{y}}$	Force of drag (with $A = \pi r^2$): $\mathbf{F}_d = -\frac{1}{2}C_d\rho A\mathbf{v}\mathbf{v}$
Equations of motion	$ma_x = -\frac{1}{2}C_d\rho A\mathbf{v}v_x$	$ma_y = -\frac{1}{2}C_d\rho A\mathbf{v}v_y - mg$

- The *computational model* is solved numerically and assumes
 - Euler’s method [10] (to illustrate *solution verification*, next slide)
 - *code verification* [3] has been performed



Inputs	Symbol	Units	μ	σ/μ
Gravity	g	m/s ²	9.81	-
Mass	m	kg	0.2	-
Radius	r	m	0.05	-
Air Density	ρ	kg/m ³	1.28	0.05
Initial Speed	v_0	m/s	50	0.01
Launch Angle	α	deg	45	0.01
Drag Coefficient	C_d	-	0.47	0.1

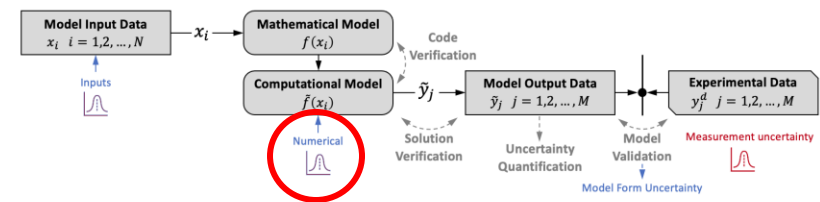
Sources of Uncertainty



Objective: Characterize uncertainty in the **model inputs** and **measured range** (σ_d)

- **Input uncertainty:** Derived from various sources including the experimental set up, specifications, or expert opinion. This uncertainty causes each launch to be slightly different, even when the setup is nominally the same.
- The projectile problem is defined with four sources of input uncertainty
 - Air density, ρ , varies during operation by $\pm 5\%$
 - Drag coefficient, C_d , is known to within $\pm 10\%$
 - Initial speed, v_0 , uncertainty due to launch mechanism setting accuracy ($\pm 1\%$)
 - Launch angle, α , uncertainty due to angle setting accuracy ($\pm 1\%$)
- **Experimental uncertainty in the measured range:** The experiment is repeated 10 times, resulting in 10 samples from which are computed the sample mean and sample standard deviation
 - Sample mean of the projectile range, $\bar{y}_d = 80.26 \text{ m}$
 - Sample standard deviation of the projectile range, $\sigma_d = \sqrt{\frac{1}{10-1} \sum_{i=1}^{10} (y_{di} - \bar{y}_d)^2} = 2.01 \text{ m}$

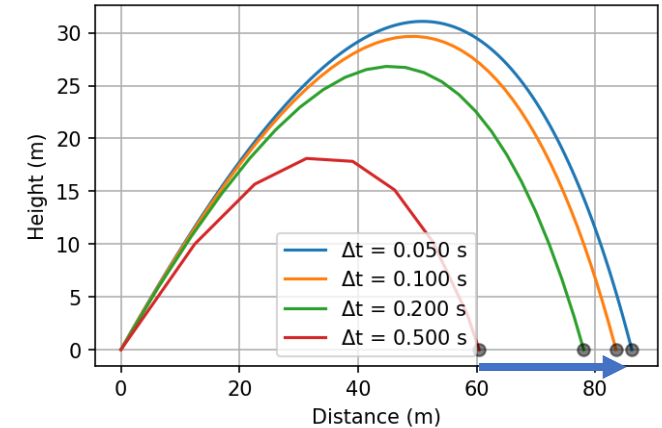
Solution Verification



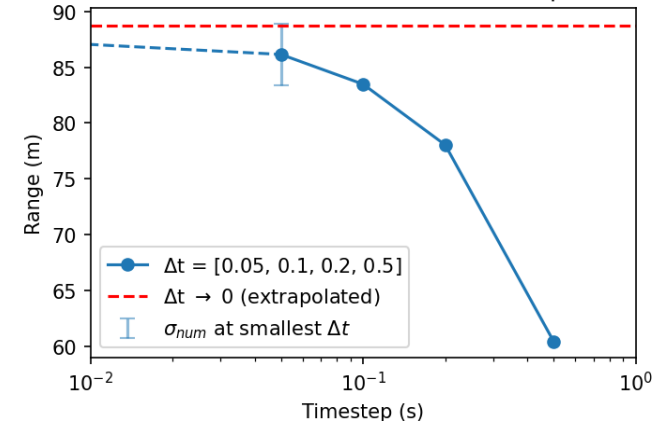
Objective: Solution verification estimates the numerical error (σ_{num}) of the output quantity of interest.

- This example solves the ODE time integration using the Euler method*, at time steps $\Delta t = [0.5, 0.2, 0.1, 0.05]$ sec.
 - $y_{n+1} = y_n + \Delta t \cdot f(t_n, y_n)$
 where, $x_{n+1} = t_n + \Delta t$
 and $f(\cdot)$ is a function of the derivative
- In many real-world problems, numerical refinement may be limited due to the computational time/cost.
 - In this example, assume the limitation $\Delta t \geq 0.05$ sec.
- Then, compute multiple solutions and perform solution verification using Richardson Extrapolation** [3] which,
 - Extrapolates to $\Delta t \rightarrow 0$ sec,
 - Estimates error of at the smallest $\Delta t = 0.05$ sec.
 - The result is $\sigma_{num} \approx 2.78$ m (shown as error bars).

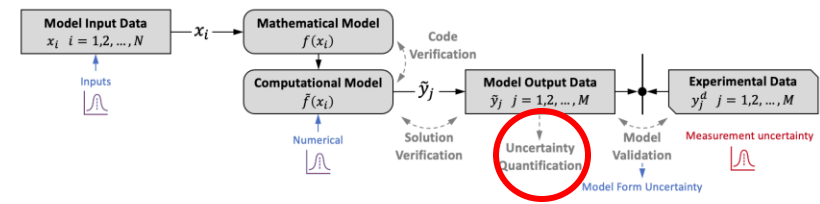
Projectile Motion with Air Resistance: Solution Verification



Solution Verification for Euler Timestep Size

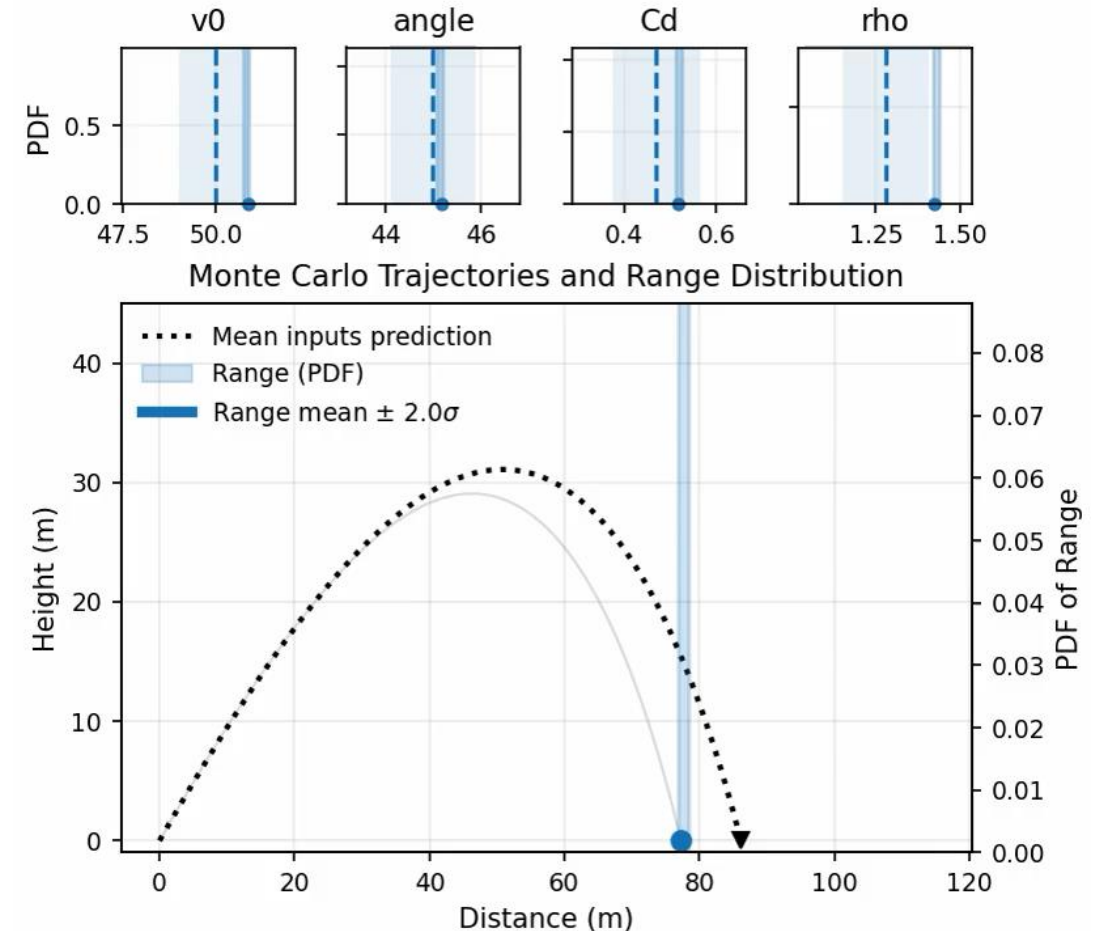


Uncertainty Quantification

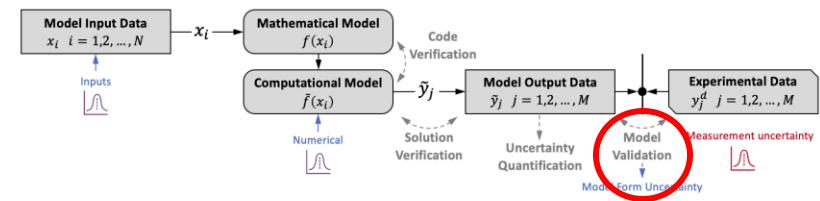


Objective: Assess how uncertainty of the model inputs (σ_{in}) affect the model outputs.

- Step 1: Characterize Input Uncertainty
 - Discussed on the Measurement Uncertainty slide
 - Draw random samples from each input distribution
- Step 2: Propagate Input Uncertainty
 - Run the model for each set of input samples to produce 10,000 possible trajectories
 - Collect the QoI (range) from each trajectory
- Step 3: Summarize the Results for the QoI
 - Compute the mean* and standard deviation** of range to summarize model uncertainty
 - Mean $\rightarrow \bar{y} = 86.61 \text{ m}$
 - Std Dev. $\rightarrow \sigma_{in} \approx 5.89 \text{ m}$
- Step 4: Summarize model uncertainty (next slide)



Validation Assessment



Objective: Quantitative comparison of predictions (y) and measured data (y_d) to assess model accuracy.

Summarize model uncertainty

- Total model uncertainty σ_{mdl} includes σ_{num} and σ_{in}
- For this example, it is assumed* that,

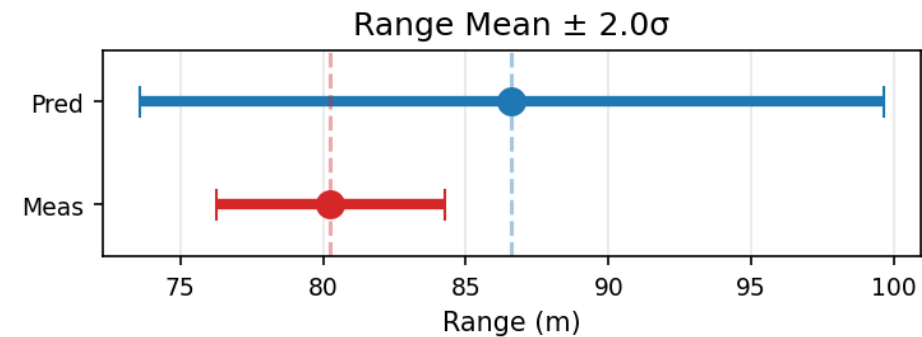
$$\sigma_{mdl} \approx \sqrt{\sigma_{num}^2 + \sigma_{in}^2} = 6.52 \text{ m.}$$

- Now, resulting model and measurement uncertainty may be summarized:

	\bar{y}	\bar{y}_d	$\delta = \bar{y} - \bar{y}_d$	σ_{mdl}	σ_d
Range, m	86.61	80.26	6.35	6.52	2.01

Perform validation assessment

- Compare y and y_d as intervals $\rightarrow \bar{y} \pm 2\sigma$
- Consider model bias** and relative uncertainty
 - Bias: $\delta = \bar{y} - \bar{y}_d \rightarrow$ model overpredicts by 6.35 m
 - $\sigma_{mdl} > \sigma_d \rightarrow$ model has greater uncertainty



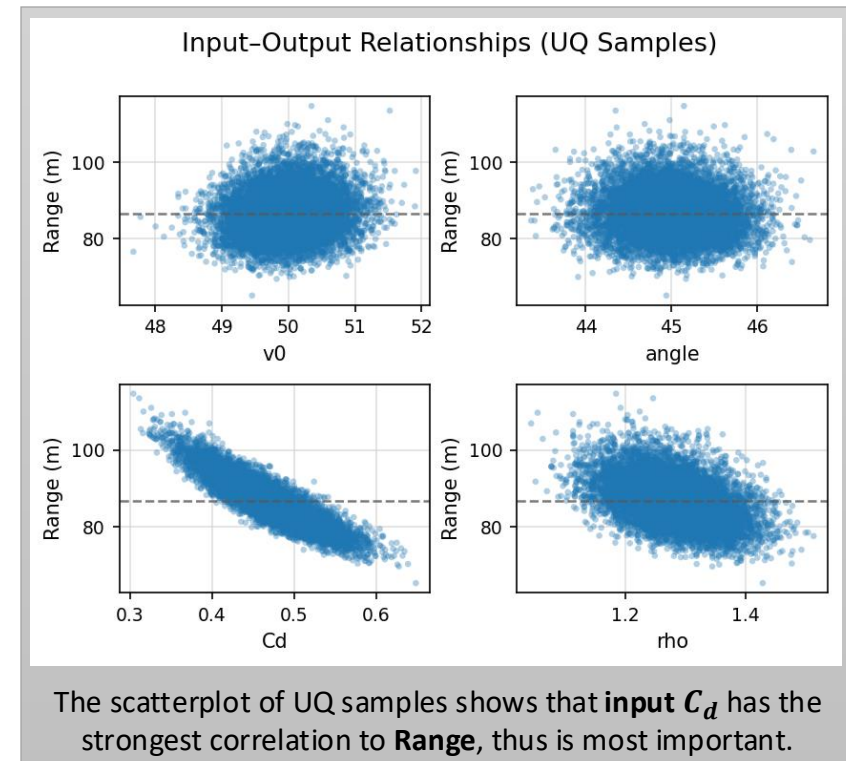
Decision-Making

Objective: Determine whether model accuracy is adequate for the intended use of the model.

- **Evaluate Requirement:** Suppose that for prediction y under similar conditions to the test, with bias $\delta = \bar{y} - \bar{y}_d$, that the prediction bound requirement is $B = 7 \text{ m}$ for $\bar{y} \pm B$ and is defined*:

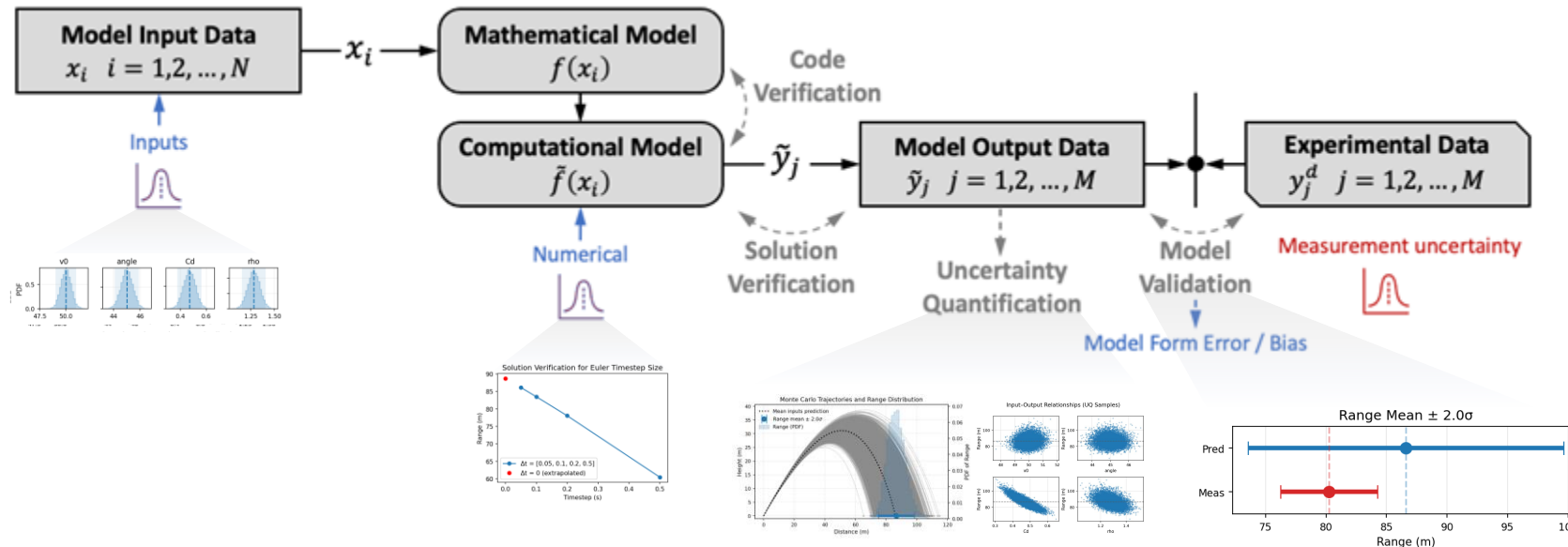
$$B = \sqrt{\sigma_d^2 + \sigma_{num}^2 + \sigma_{in}^2} + |\delta| \rightarrow \text{Result: } B \approx \pm 13.1 \text{ m}$$

- **Decision:** Uncertainty reduction is required
 - Determine which uncertainty source to reduce:
 - $\sigma_d = 2.01 \text{ m}$ → more expensive instrumentation
 - $\sigma_{num} = 2.78 \text{ m}$ → affects computational time/cost
 - $\sigma_{in} = 5.89 \text{ m}$ → involves model improvements
 - σ_{in} is the largest and C_d is the key driving input (see scatterplot →)
- **Next steps:** Perform the following tasks
 - Refine C_d to reduce σ_{in} to <3 m
 - Refine time-step accuracy to reduce σ_{num} to <1 m
 - Reevaluate the magnitude of bias after these improvements



Conclusions

- **VVUQ links models to decisions:** define the conceptual/mathematical model, solve the numerical model, quantify uncertainty, compare to data, and judge credibility for intended use.
- As engineering increasingly relies on physics-based models and derivative ML/AI-enabled tools, engineers must understand **how trustworthy it is for the decision being made.**
- Using the simple projectile motion example, we demonstrated how the VVUQ process enables the **collection of evidence to support the credibility of the model** for its intended use.



References

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Additional Information (optional)

- Ref. [1] expands on this example to consider the various decisions the analyst must make when performing VVUQ
- **Model form error** arises when the model does not represent the desired use-case
 - May be partially quantified during validation if the experiment represents the ‘real-world’ use-case
 - For example, the projectile model may have model form error due to unaccounted for wind, spin on the ball ([Magnus effect](#)), drag assumption errors (roughness, shape), density differences due to temperature/altitude
- Considerations for estimation of σ_{in} in more complex problems
 - Reliable estimation relies on correct characterization of input uncertainty AND low model form error
 - Justification for the input uncertainty definitions are one of the most important considerations for UQ!
 - Some inputs require significant data collection efforts and careful statistical analysis
 - **Bayesian inference** may be necessary to quantify probability distribution parameters (e.g., due to limited sample size) and some physics model parameters may require estimation using advanced methods such as **Bayesian inference**
 - **Sensitivity analysis** is a means to identifying the most important inputs – crucial for reducing problem size
 - Propagation using Monte Carlo simulation is often cost prohibitive for complex models and may require the use of Design of Experiments (DoE), surrogate models, and high-performance computing
- Obtaining sufficient test data to fully characterize σ_d is often infeasible. This is in part because the outputs of interest require system level testing. Estimation of this quantity often requires approximation and expert judgment.
- Determination of σ_{num} through solution verification is challenging in many real-world applications
 - Solution verification involves refinement of spatial and temporal dimensions and may involve many quantities of interest
 - May involve irregular (unstructured) spatial grids that cannot be refined with systematic uniform refinement.
- Validation assessment is a broad topic and there are many ways people consider this, including many metric types, e.g., [3,6,11].
- The above considerations show that proper application of VVUQ requires appropriate methods, tools, and training – all of these are on-going areas of research.