

APPENDIX D: SAMPLE OF UNCERTAINTY ANALYSIS

Pioro, I.L. and Duffey, R.B., 2007. Heat Transfer and Hydraulic Resistance at Supercritical Pressures in Power Engineering Applications, ASME Press, New York, NY, USA, 334 pages.

The proposed uncertainty analysis¹ is based on our current experience with heat-transfer and pressure-drop experiments in supercritical water (Kirillov et al. 2005; Pis'menny et al. 2005) and carbon dioxide (Pioro and Khartabil 2005) and on our long-term experience in conducting heat-transfer experiments at subcritical pressures (Guo et al. 2006; Bezrodny et al. 2005; Leung et al. 2003; Pioro et al. 2002a,b, 2001, 2000; Pioro 1999, 1992; Pioro and Pioro 1997; Kichigin and Pioro 1992; Pioro and Kalashnikov 1988; Pioro 1982). Also, basic principles of the theory of thermophysical experiments and their uncertainties were applied (Coleman and Steel 1999; Hardy et al. 1999; Guide... 1995; Holman 1994; Moffat 1988; Gortyshov et al. 1985; Topping 1971).

In general, an uncertainty analysis is quite complicated process in which some uncertainties² (for example, uncertainties of thermophysical properties (for details, see NIST (2002)), uncertainties of constants, etc.) may not be known or may not be exactly calculated. Therefore, applying the engineering judgement is the only choice in some uncertainty calculations.

This section summarizes instrument calibrations and uncertainty calculations for the measured parameters such as temperature, pressure, pressure drop, mass-flow rate, power, tube dimensions, etc. and for the calculated parameters such as mass flux, heat flux, etc. in supercritical heat-transfer and pressure-drop tests. Uncertainties for these parameters are based on the RMS of component uncertainties. All uncertainty values are at the 2σ level, unless otherwise specified.

¹ The authors of the current monograph express their appreciation to D. Bullock and Y. Lachance (CRL AECL) for their help in preparation of this uncertainty analysis.

² Uncertainty refers to the accuracy of measurement standards and equals the sum of the errors that are at work to make the measured value different from the true value. The accuracy of an instrument is the closeness with which its reading approaches the true value of the variable being measured. Accuracy is commonly expressed as a percentage of a measurement span, measurement value or full-span value. Span is the difference between the full-scale and the zero scale value (Mark's Standard Handbook for Mechanical Engineers 1996).

Calibration of the instruments used in the tests was performed either *in situ*, e.g., power measurements, test-section thermocouples, etc., or at an instrumentation shop, e.g., pressure transducers and bulk-fluid temperature thermocouples. In general, instruments were tested against a corresponding calibration standard.

When the same calibration standard is used for serial instruments, the calibration standard uncertainty is treated as a systematic uncertainty. In general, high accuracy calibrators were used, hence systematic errors for calibrated instruments are considered to be negligible. All other uncertainties are assumed to be random. Also, errors correspond to the normal distribution. Usually, the uncertainties have to be evaluated for three values of the corresponding parameter: minimum, mean and maximum value within the investigated range.

Uncertainties are presented below for instruments, which are commonly used in heat-transfer and pressure-drop experiments. It is important to know the exact schematics for sensor signal processing. Some commonly used cases, which are mainly based on a DAS recording, are shown in Figure D1 for thermocouples and in Figure D2 for RTDs, pressure cells and differential pressure cells.

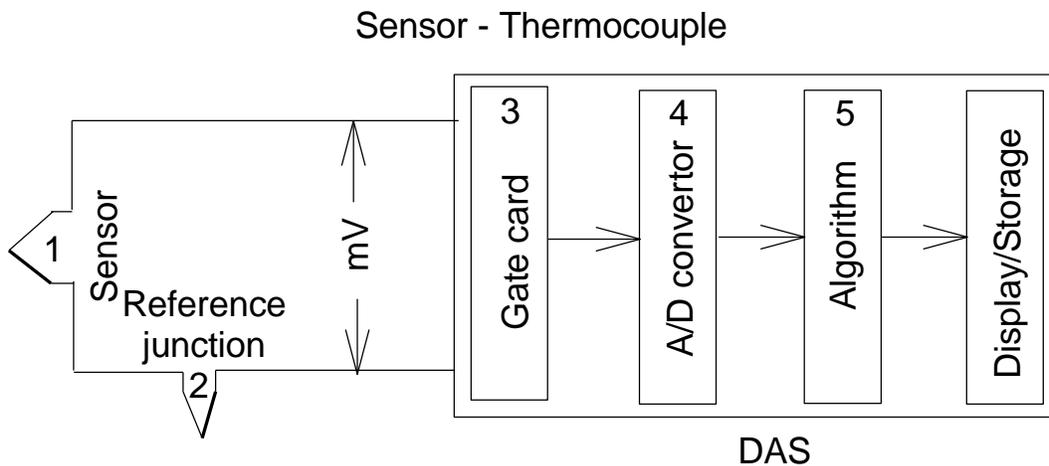


Figure D1. Schematic of signal processing for temperature (based on thermocouple) measurements. Numbers in figure identify uncertainty of particular device in measuring circuit: 1 – sensor uncertainty, 2 – reference junction uncertainty, 3 – Analog Input (A/I) uncertainty, 4 – Analog-to-Digital (A/D) conversion uncertainty, and 5 – DAS algorithm uncertainty.

Sensor - RTD, Pressure cell and Differential Pressure Cell

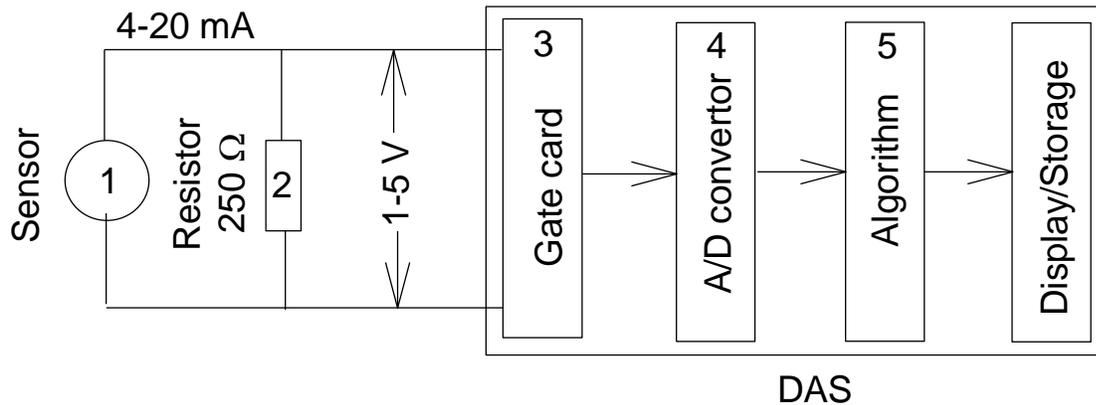


Figure D2. Schematic of signal processing for temperature (based on RTD), absolute pressure and differential pressure. Numbers in figure identify uncertainty of particular device in measuring circuit: 1 – sensor uncertainty, 2 – uncertainty due to temperature effect, 3 – A/I uncertainty, 4 – A/D conversion uncertainty, and 5 – DAS algorithm uncertainty; for RTD and both types pressure cells – DAS algorithm uncertainty is usually 0 due to linear fit.

Also, absolute and relative errors for commonly used functions are listed in Table D1 for reference purposes.

Table D1. Absolute and relative errors for commonly used functions (based on Gortyshov et al. (1985)).

Function	Absolute error	Relative error
$Y = X_1 + X_2 + \dots + X_n$	$\pm \sqrt{\Delta X_1^2 + \Delta X_2^2 + \dots + \Delta X_n^2}$	$\pm \frac{\sqrt{\Delta X_1^2 + \Delta X_2^2 + \dots + \Delta X_n^2}}{X_1 + X_2 + \dots + X_n}$
$Y = \frac{X_1 + X_2 + \dots + X_n}{n}$	$\pm \frac{\sqrt{\Delta X_1^2 + \Delta X_2^2 + \dots + \Delta X_n^2}}{\sqrt{n}}$	$\pm \frac{\sqrt{n} \cdot \sqrt{\Delta X_1^2 + \Delta X_2^2 + \dots + \Delta X_n^2}}{X_1 + X_2 + \dots + X_n}$
$Y = X_1 - X_2$	$\pm \sqrt{\Delta X_1^2 + \Delta X_2^2}$	$\pm \frac{\sqrt{\Delta X_1^2 + \Delta X_2^2}}{X_1 - X_2}$

$Y = X_1 \cdot X_2$	$\pm \sqrt{(X_1 \cdot \Delta X_2)^2 + (X_2 \cdot \Delta X_1)^2}$	$\pm \sqrt{\left(\frac{\Delta X_1}{X_1}\right)^2 + \left(\frac{\Delta X_2}{X_2}\right)^2}$
$Y = a \cdot X$	$\pm a \cdot \Delta X$	$\pm \frac{\Delta X}{X}$
$Y = X^n$	$\pm n \cdot X^{n-1} \Delta X$	$\pm \frac{n \cdot \Delta X}{X}$
$Y = \sin X$	$\pm \cos X \cdot \Delta X$	$\pm \text{ctg}X \cdot \Delta X$
$Y = \cos X$	$\pm \sin X \cdot \Delta X$	$\pm \text{tg}X \cdot \Delta X$
$Y = \text{tg}X$	$\pm \frac{\Delta X}{\cos^2 X}$	$\pm \frac{2 \cdot \Delta X}{\sin(2 \cdot X)}$
$Y = \text{ctg}X$	$\pm \frac{\Delta X}{\sin^2 X}$	$\pm \frac{2 \cdot \Delta X}{\sin(2 \cdot X)}$

D.1 Temperature

For the calibrated thermocouples, the following linear characteristics were found:

$$V_{act} = a \cdot V_{meas} + b, \quad (\text{D1})$$

where V_{act} is the “actual” value³ of the given parameter, V_{meas} is the value measured by the given instrument, and a and b are the calibration coefficients.

D.1.1 Measured Bulk-Fluid Temperature

The test section (see Figures 10.5 and 10.6) has three thermocouples to measure the inlet and outlet bulk-fluid temperatures. Also, the temperature at the flowmeter is monitored by thermocouple for fluid density calculations.

The test-section inlet and outlet bulk-fluid temperatures were measured with sheathed K-type thermocouples (for thermocouple signal processing, see Figure D1). These thermocouples

³ The value obtained from the calibration standard.

were calibrated against the temperature standard RTD over the temperature range from 0 to 100°C. For the reference RTD, the maximum error was $\pm 0.3^\circ\text{C}$. The maximum uncertainty of a data fit for inlet and outlet bulk-fluid temperature measurements is listed in Table D2.

Table D2. Linear coefficients for inlet and outlet temperature thermocouples (from instrument calibration records).

TC	Coefficient		Uncertainty, °C	Number of points
	<i>a</i>	<i>b</i>	Maximum (2σ)	
–				–
TE-1	1.000	–0.1798	0.12	5
TE-2	0.9980	0.1502	0.12	5
TE-3	0.9985	0.0980	0.12	5

The inlet and outlet bulk-fluid measurement uncertainties⁴ are as follows:

Calibration system uncertainty	$\pm 0.3^\circ\text{C}$
Thermocouple sensor accuracy after linear fit	$\pm 0.12^\circ\text{C}$
A/I accuracy	$\pm 0.06^\circ\text{C}$, i.e., $\pm 0.025\%$ of f.s.; $\left(= \pm \frac{0.00025 \cdot 10 \text{ mV}}{0.045 \frac{\text{mV}}{^\circ\text{C}}} \right)$; where f.s. is the full scale
A/D resolution accuracy (minimum 1 bit)	$\pm 0.03^\circ\text{C} \left(= \pm \frac{10 \text{ mV (f.s.)}}{8192 \text{ counts} \cdot 0.045 \frac{\text{mV}}{^\circ\text{C}}} \right)$, where 0.045 mV/°C is the conversion rate, i.e., 4.509 mV for 100°C (The Temperature Handbook 2000)
Reference junction	$\pm 0.4^\circ\text{C}$

⁴ All inputs are from instrument calibration records and device manuals unless otherwise specified.

accuracy	
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For a given test-section inlet or outlet temperature t , the uncertainty Δt is given by

$$\frac{\Delta t}{t} = \sqrt{\left(\frac{0.3}{t}\right)^2 + \left(\frac{0.12}{t}\right)^2 + \left(\frac{0.03}{t}\right)^2} \quad (D2)$$

The first term is the maximum error of the calibration system ($\pm 0.3^\circ\text{C}$). The second term is the maximum error for the sheathed thermocouple ($\leq 100^\circ\text{C}$), obtained from the calibration. The third term is the uncertainty introduced by the DAS, i.e., the A/D resolution uncertainty ($\pm 0.03^\circ\text{C}$). Note that since the calibration was done *in situ* using the DAS as the measuring system for the RTD and for the calibrated thermocouples, the uncertainty introduced by the reference junction and the A/I accuracy was included in calibration curves.

All bulk-fluid temperature thermocouples were calibrated *in situ*, only within the range of 0 – 100°C. Therefore, individual correction factors were implemented for each thermocouple within the range of 0 – 100°C (see Table D2). For this range of temperatures, the uncertainty Δt is

- for $t_{min} = 20^\circ\text{C}$ $\Delta t = \pm 0.32^\circ\text{C}$ (or $\pm 1.62\%$), and
- for $t = 100^\circ\text{C}$ $\Delta t = \pm 0.32^\circ\text{C}$ (or $\pm 0.32\%$).

Beyond this range, thermocouple uncertainties were taken as per The Temperature Handbook (2000), i.e., $\pm 2.2^\circ\text{C}$.

Thermocouple installed near the flowmeter was calibrated using another calibrating system and procedure. All inputs below are from instrument calibration record and device manuals unless otherwise specified.

Calibration system uncertainty:

$\pm 0.5^\circ\text{C}$, i.e., $\left(= \pm \sqrt{0.06^2 + 0.5^2 + 0.041^2}\right)$, where the first term is the accuracy of standard RTD, the second term is the accuracy of thermocouple signal measuring device and the third term is the accuracy of RTD signal measuring device (all uncertainties are in $^\circ\text{C}$).

TC maximum calibration	$\pm 0.53^\circ\text{C}$
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accuracy ⁵ ($>2\sigma$) within 0.0 – 45.0°C	
A/I accuracy	$\pm 0.06^\circ\text{C}$, i.e., $\pm 0.025\%$ of f.s. $\left(= \pm \frac{0.00025 \cdot 10 \text{ mV}}{0.045 \frac{\text{mV}}{^\circ\text{C}}} \right)$
A/D resolution accuracy (minimum 1 bit)	$\pm 0.03^\circ\text{C} \left(= \pm \frac{10 \text{ mV (f.s.)}}{8192 \text{ counts} \cdot 0.045 \frac{\text{mV}}{^\circ\text{C}}} \right)$, where 0.045 mV/°C is the conversion rate, i.e., 4.509 mV for 100°C (The Temperature Handbook 2000)
Reference junction accuracy	$\pm 0.02^\circ\text{C}$

For a given flowmeter bulk-fluid temperature t_{fm} , the uncertainty Δt_{fm} is given by

$$\left(\frac{\Delta t_{fm}}{t_{fm}} \right)_{TC} = \left(\pm \sqrt{\left(\frac{0.5}{t_{fm}} \right)^2 + \left(\frac{0.53}{t_{fm}} \right)^2 + \left(\frac{0.06}{t_{out}} \right)^2 + \left(\frac{0.03}{t_{out}} \right)^2 + \left(\frac{0.02}{t_{out}} \right)^2} \right)_{TC}. \quad (\text{D3})$$

Therefore, the flowmeter bulk-fluid temperature uncertainty is:

- for $t_{fm \min} = 19^\circ\text{C}$ $\Delta t_{fm} = \pm 0.74^\circ\text{C}$ (or $\pm 3.9\%$), and
- for $t_{fm \max} = 35^\circ\text{C}$ $\Delta t_{fm} = \pm 0.74^\circ\text{C}$ (or $\pm 2.1\%$).

Additional uncertainties due to thermocouple installation and possible electrical pickup have been minimized by using good engineering practices.

If a bulk-fluid temperature is measured with an RTD, then the following will apply.

The bulk-fluid temperature measurement uncertainties at the 2σ level are characterized with the following for an RTD (for RTD signal processing, see Figure D2):

⁵ The TC calibration accuracy is the maximum difference in °C between what the calibration standard measured and what TC indicated.

Calibration system uncertainty in °C (from the instrument calibration record):

$$\text{Cal. Sys. Unc.} = \pm \sqrt{\left[\left(0.01\% \text{ of Reading } \frac{16 \text{ mA}}{100^\circ\text{C}} + 0.015\% \text{ of f.s.} \right) \frac{100^\circ\text{C}}{16 \text{ mA}} \right]^2 + (0.05^\circ\text{C})^2} \approx 0.06^\circ\text{C},$$

where the first term is the accuracy of calibrator in which reading is in °C and f.s. is 30 mA and a conversion rate is 16 mA for 100°C; and the second term is the accuracy of standard RTD.

The RTD accuracy after linear fit, i.e., maximum deviation (from the instrument calibration record), is about $\pm 0.08^\circ\text{C}$;

A/I accuracy (from the device manual):

$$\pm 0.032^\circ\text{C} (\pm 0.025\% \text{ of f.s.}), \text{ i.e., } \left(= \pm \frac{0.00025 \cdot 5.12 \text{ V (f.s.)}}{0.04 \frac{\text{V}}{^\circ\text{C}}} \right).$$

A/D conversion accuracy (minimum 1 bit accuracy) (from the device manual):

$$\pm 0.016^\circ\text{C}, \text{ i.e., } \left(= \pm \frac{5.12 \text{ V (f.s.)}}{8192 \text{ counts} \cdot 0.04 \frac{\text{V}}{^\circ\text{C}}} \right), \text{ where } 0.04 \text{ V}/^\circ\text{C} \text{ is the conversion rate, i.e., } 4 \text{ V for}$$

100°C (from the instrument calibration record).

DAS algorithm uncertainty is 0 due to a linear fit.

Therefore, for a given test-section inlet temperature, its uncertainty (Δt) is given by

$$\frac{\Delta t}{t} = \pm \sqrt{\left(\frac{\text{Cal. Sys. Unc.}}{t} \right)^2 + \left(\frac{0.08}{t} \right)^2 + \left(\frac{0.032}{t} \right)^2 + \left(\frac{0.016}{t} \right)^2}. \quad (\text{D4})$$

The resulting uncertainties in the bulk-fluid temperature are

- For $t = 10^\circ\text{C}$ $\Delta t = \pm 0.10^\circ\text{C}$ (or $\pm 1.2\%$); and
- For $t = 50^\circ\text{C}$ $\Delta t = \pm 0.11^\circ\text{C}$ (or $\pm 0.2\%$).

If the bulk-fluid temperature is measured with several devices installed in a one cross section (for example, two RTDs and one thermocouple), the following equation may apply:

$$\frac{\Delta \bar{t}}{\bar{t}} \cong \pm \frac{\sqrt{(\Delta t_{RTD1})^2 + (\Delta t_{RTD2})^2 + (\Delta t_{TC})^2}}{\sqrt{3} \bar{t}} \quad (D5)$$

In this case, the resulting uncertainty will be close to the larger uncertainty, i.e., the thermocouple uncertainty. Therefore, if several devices have to be used for measuring a non-uniform temperature or any other parameter, they have to be with a similar accuracy.

D.1.2 External Wall Temperature

Temperatures for the test-section external surface (see Figure 10.6) were measured using fast-response K-type thermocouples (see Figure D3). In general, thermocouple uncertainties for K-type thermocouples are $\pm 2.2^\circ\text{C}$ within a range of 0 – 277°C (The Temperature Handbook, 2000). However, all fast-response thermocouples were calibrated *in situ* within a range of 0 – 100°C prior to use (for details, see below). Therefore, individual correction factors were implemented for each thermocouple within the range of 0 – 100°C. Beyond this range, thermocouple uncertainties were taken as per The Temperature Handbook (2000), i.e., $\pm 2.2^\circ\text{C}$.

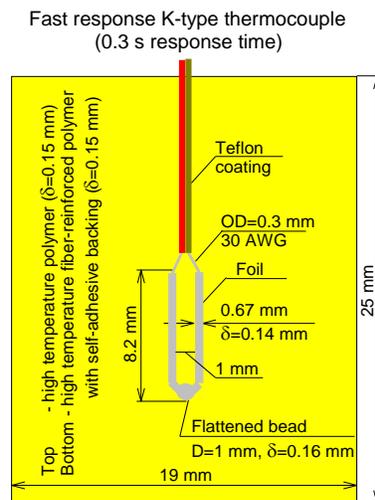


Figure D3. Sketch drawing of fast-response K-type thermocouple.

All K-type thermocouples were calibrated against the temperature calibration standard (i.e., the reference RTD) over the temperature range from 0 to 100°C. These thermocouple

assemblies were immersed in a liquid bath thermostat together with the RTD. For the reference RTD, the maximum uncertainty is $\pm 0.3^\circ\text{C}$. The combined uncertainty⁶ for wall temperature measurements is as follows:

Calibration system accuracy	$\pm 0.3^\circ\text{C}$
Thermocouple sensor accuracy after linear fit	$\pm 0.16^\circ\text{C}$ max at values $\leq 100^\circ\text{C}$
A/I accuracy	$\pm 0.06^\circ\text{C}$, i.e., $\pm 0.025\%$ of f.s. $\left(= \pm \frac{0.00025 \cdot 10 \text{ mV}}{0.045 \frac{\text{mV}}{^\circ\text{C}}} \right)$
A/D resolution accuracy (minimum 1 bit)	$\pm 0.03^\circ\text{C} \left(= \pm \frac{10 \text{ mV (f.s.)}}{8192 \text{ counts} \cdot 0.045 \frac{\text{mV}}{^\circ\text{C}}} \right)$, where 0.045 mV/ $^\circ\text{C}$ is the conversion rate, i.e., 4.509 mV for 100°C (The Temperature Handbook 2000)

For a given test-section wall temperature t , the uncertainty Δt is given by

$$\frac{\Delta t}{t} = \sqrt{\left(\frac{0.3}{t}\right)^2 + \left(\frac{0.16}{t}\right)^2 + \left(\frac{0.03}{t}\right)^2}. \quad (\text{D6})$$

The first term is the maximum error of the calibration system ($\pm 0.3^\circ\text{C}$). The second term is the maximum error of the sheathed thermocouple ($\leq 100^\circ\text{C}$), obtained from the calibration. The third term is the uncertainty introduced by the DAS, i.e., the A/D resolution uncertainty ($\pm 0.03^\circ\text{C}$). Note that since the calibration was done *in situ* using the DAS as the measuring system for the RTD and the calibrated thermocouples, the uncertainty introduced by the reference junction and the A/I accuracy was included in calibration curves.

Within the calibrated range of measured temperatures, i.e., from 0 to 100°C , the

⁶ All inputs are from instrument calibration records and device manuals unless otherwise specified.

uncertainty Δt is

- for $t_{min} = 25^\circ\text{C}$ $\Delta t = \pm 0.34^\circ\text{C}$ (or $\pm 1.36\%$), and
- for $t = 100^\circ\text{C}$ $\Delta t = \pm 0.34^\circ\text{C}$ (or $\pm 0.34\%$).

Also, the external wall temperatures measured with fast-response thermocouples were compared to the inlet and outlet bulk-fluid temperatures measured with sheathed thermocouples, at 0 power and 0 mass flux through the test section (see Figure D4). The comparison showed that, in general, all measured temperatures were within $\pm 0.3^\circ\text{C}$.

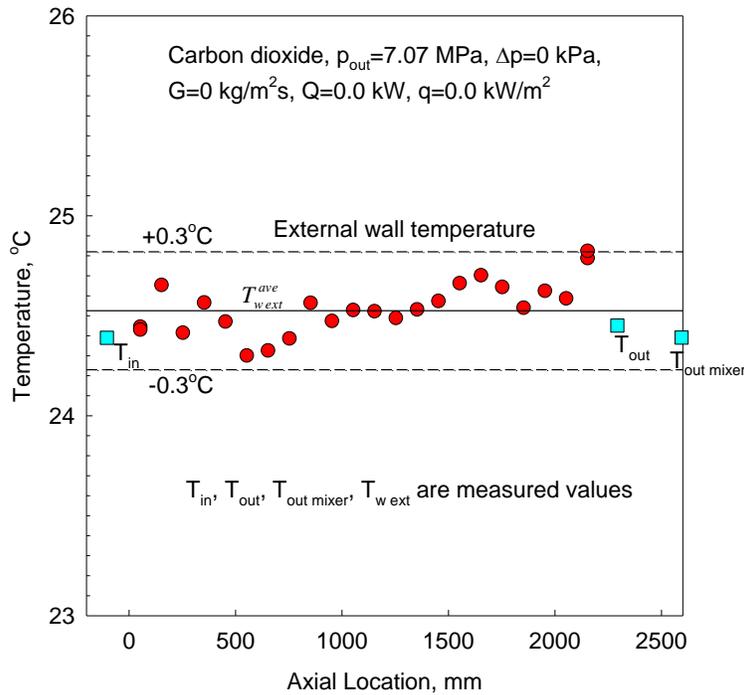


Figure D4. Temperature profile along test section at 0 power and 0 mass flux values.

D.2 Absolute Pressure

A high-accuracy gauge pressure cell with a range of 0 – 10,000 kPa (0 – 10 MPa) was used for the outlet-pressure measurements (for pressure signal processing, see Figure D2). A small correction (77.2 kPa) is applied in the DAS program for the elevation difference between the pressure tap and transmitter. The combined uncertainty for absolute pressure measurements is as follows.

Accuracy of gauge pressure cell (from the calibration record) is $\pm 0.1\%$ of calibrated span (10,000 kPa), and this accuracy was verified during the calibration check.

Calibration system uncertainty in kPa (from the instrument calibration record):

Cal. Sys. Unc. = $\pm \sqrt{\left[\left(0.015\% \text{ of Reading } \frac{16 \text{ mA}}{10000 \text{ kPa}} + 0.015\% \text{ of f.s.} \right) \frac{10000 \text{ kPa}}{16 \text{ mA}} \right]^2 + (0.1\% \text{ of Reading})^2}$, where the first term is the accuracy of calibrator in which reading is in kPa and f.s. is 30 mA and conversion rate is 16 mA for 10000 kPa; and the second term is the accuracy of tester.

Uncertainty due to temperature effect in 250- Ω resistor:

$\pm 0.1\%$.

A/I accuracy (from the device manual):

$$\pm 3.2 \text{ kPa, i.e., } \pm 0.025\% \text{ of f.s., i.e., } 5.12 \text{ V} \left(= \pm \frac{0.00025 \cdot 5.12 \text{ V}}{0.0004 \frac{\text{V}}{\text{kPa}}} \right).$$

A/D conversion accuracy (minimum 1 bit accuracy) (from the device manual):

$$\pm 1.56 \text{ kPa} \left(= \pm \frac{5.12 \text{ V (f.s.)}}{8192 \text{ counts} \cdot 0.0004 \frac{\text{V}}{\text{kPa}}} \right), \text{ where } 0.0004 \text{ V/kPa is the conversion rate, i.e., } 4 \text{ V}$$

for 10,000 kPa (from the instrument calibration record).

For a given test-section outlet pressure p , the uncertainty Δp is given by

$$\frac{\Delta p}{p} = \pm \sqrt{\left(\frac{0.001 \cdot 10000}{p} \right)^2 + \left(\frac{0.1}{100} \right)^2 + \left(\frac{A/I}{p} \right)^2 + \left(\frac{A/D}{p} \right)^2}, \quad (\text{D7})$$

For the range of p from 7.6 to 8.8 MPa, the uncertainty Δp is given by

- for $p_{min} = 7600 \text{ kPa}$ $\Delta p = \pm 13.1 \text{ kPa (or } \pm 0.17\%),$
- for $p = 8400 \text{ kPa}$ $\Delta p = \pm 13.5 \text{ kPa (or } \pm 0.16\%).$
- for $p_{max} = 8800 \text{ kPa}$ $\Delta p = \pm 13.8 \text{ kPa (or } \pm 0.16\%).$

D.3 Differential-Pressure Cells

Five differential-pressure transducers for measuring test-section pressure drops (for differential-pressure signal processing, see Figure D2) were connected to the corresponding pressure taps installed as shown in Figure 10.6. They were used for measuring the test-section axial pressure gradient and the overall pressure drop. Also, one differential-pressure transducer was used to measure a pressure drop across the flowmeter (see Figure 10.5). All these pressure drops were measured using pressure transmitters.

A calibrator and a pressure module were used for the calibration check of the differential-pressure transducers. Basic characteristics of the test-section and flowmeter differential-pressure cells are listed in Table D3.

Table D3. Basic characteristics of differential-pressure cells.

Instrument Name	Description	Output	Output kPa	Span kPa	Accuracy ±% of span
PDT-1	Total test-section pressure drop	10–50 mV	0–300	300	0.5
PDT-2	Test-section pressure drop	1–5 V	0–50	50	0.5
PDT-3	Test-section pressure drop	1–5 V	0–50	50	0.5
PDT-4	Test-section pressure drop	1–5 V	0–50	50	0.5
PDT-5	Test-section pressure drop	1–5 V	0–50	50	0.5
PDT-FM-1	Orifice-plate pressure drop	10–50 mV	0–37	37	0.5

Accuracy, includes combined effects of linearity, hysteresis and repeatability in % of a calibrated span are listed in Table D3.

Calibration system uncertainty in kPa (from instrument calibration records):

$$\text{Cal. Sys. Unc.} = \pm \sqrt{\left[\left(0.015\% \text{ of Reading } \frac{16 \text{ mA}}{\text{Span kPa}} + 0.015\% \text{ of f.s.} \right) \frac{\text{Span kPa}}{16 \text{ mA}} \right]^2 + (0.05\% \text{ of f.s.})^2}, \text{ where the}$$

first term is the accuracy of process calibrator in which reading is in kPa, f.s. is 30 mA and conversion rate is 16 mA for span in kPa; and the second term is the accuracy of calibrator in which f.s. is 690 kPa (100 psig).

Uncertainty due to temperature effect in 250-Ω resistor:

±0.1%.

A/I accuracy (from a device manual):

$$\pm 0.025\% \text{ of f.s., i.e., } \left(= \pm \frac{0.00025 \cdot 5.12 \text{ V}}{\frac{4 \text{ V}}{\text{Span kPa}}} \right).$$

A/D conversion accuracy (minimum 1 bit accuracy) (from a device manual):

$$\left(= \pm \frac{5.12 \text{ V (f.s.)}}{8192 \text{ counts} \cdot \frac{4 \text{ V}}{\text{Span kPa}}} \right).$$

For a given pressure drop (Δp) for PDT-1, PDT-2 to PDT-5 and PDT-FM-1, the uncertainty $\Delta(\Delta p)$ is given by

$$\frac{\Delta(\Delta p)}{\Delta p} = \pm \sqrt{\left(\frac{\% \text{ of span in kPa}}{\Delta p} \right)^2 + \left(\frac{0.1}{100} \right)^2 + \left(\frac{A/I}{\Delta p} \right)^2 + \left(\frac{A/D}{\Delta p} \right)^2}. \quad (D8)$$

For the range of the total Δp from 5 to 70 kPa, the uncertainty $\Delta(\Delta p)$ for PDT-1 is given by

- for $\Delta p_{min} = 5 \text{ kPa}$ $\Delta(\Delta p) = \pm 1.50 \text{ kPa}$ (or ±30.1%), and
- for $\Delta p_{max} = 70 \text{ kPa}$ $\Delta(\Delta p) = \pm 1.51 \text{ kPa}$ (or ± 2.2%).

For the range of the local Δp from 5 to 30 kPa, the uncertainty $\Delta(\Delta p)$ for PDT-2 – PDT-5 is given by

- for $\Delta p_{min} = 5 \text{ kPa}$ $\Delta(\Delta p) = \pm 0.25 \text{ kPa}$ (or ±5.0%), and
- for $\Delta p_{max} = 30 \text{ kPa}$ $\Delta(\Delta p) = \pm 0.25 \text{ kPa}$ (or ±0.84%).

For the local Δp equals to 37 kPa, the uncertainty $\Delta(\Delta p)$ for PDT-FM-1 is given by

- for $\Delta p_{min} = 1.5 \text{ kPa}$ $\Delta(\Delta p) = \pm 0.19 \text{ kPa}$ (or ±12.5%), and
- for $\Delta p_{max} = 16.9 \text{ kPa}$ $\Delta(\Delta p) = \pm 0.19 \text{ kPa}$ (or ± 1.1%).

D.4 Mass-Flow Rate

The loop mass-flow rate FM-1 (see Figure 10.5) is measured by a small orifice plate⁷ with an orifice diameter of 0.308", and monitored by a differential-pressure cell with the range of 0 – 37 kPa. This cell has a square root output, with an accuracy of $\pm 0.5\%$ of full scale. The square root output is converted in the program to obtain kPa for use in the following flow equation, for a mass-flow rate of 0 – 0.24 kg/s (see Figure D4):

$$m = C_{fl} \sqrt{\rho \Delta p}, \quad (\text{D9})$$

where $C_{fl} = 0.00130$ is the constant (White 1994), ρ is the density at the orifice plate in kg/m^3 , and Δp is the pressure drop across the orifice plate in kPa. It is known that orifice-plate flowmeters usually have a working range within $(0.3 \text{ and } 1) \cdot m_{max}$, i.e., 0.08 – 0.24 kg/s (The Flow and Level Handbook 2001).

In general, the constant C_{flow} is a function of Reynolds number (see Figure D5). However, this effect is minor within the investigated range of Reynolds numbers ($\text{Re} = 57,000 - 1,130,000$).

⁷ This small diameter orifice plate is a non-standard orifice plate, because International Standard ISO 5167-2:2003(E), "Measurement of fluid flow by means of pressure differential devices inserted in circular-cross section conduits running full – Part 2: Orifice Plates", applies only to orifice plates with a diameter not less than 12.5 mm.

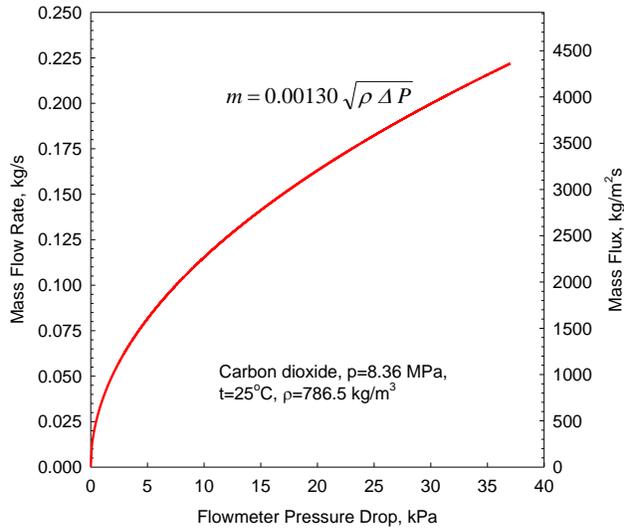


Figure D4. Flow-measurement curve.

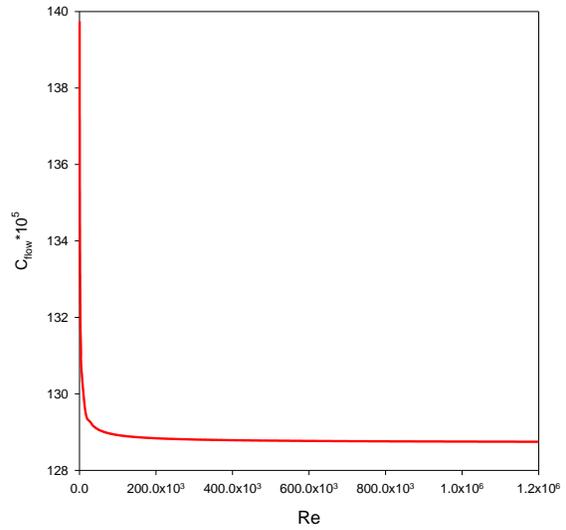
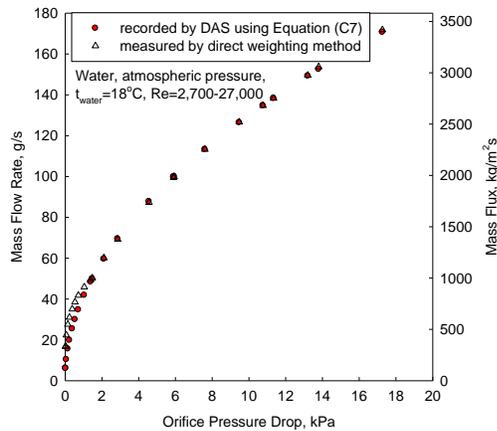
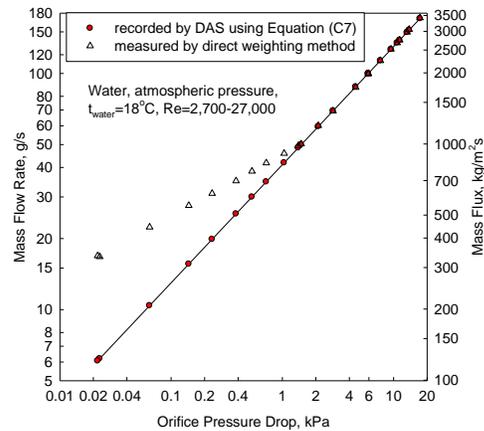


Figure D5. Effect of Reynolds number on flow constant

We attempted to calibrate the flowmeter FM-1 with water using the direct weighting method (Hardy et al. 1999) within the supercritical CO₂ investigated Reynolds numbers range. Due to significantly different values of water dynamic viscosity compared to those of supercritical carbon dioxide and restrictions applied to the maximum water flow and its temperature, the flowmeter was calibrated (see Figure D6) within a lower range of Reynolds numbers (**Re** = 2,700 – 27, 000) compared to those of supercritical carbon dioxide (**Re** = 57,000 – 1,130,000).



(a)



(b)

Figure D6. Calibration results for FM-1 orifice-plate flowmeter: (a) linear scale, and (b) logarithmic scale.

However, the calibration results showed that Equation (D9) is reasonably accurate (a mean error is -0.15% and an RMS error is 0.5%) for flows that are not less than 0.045 kg/s. This finding is consistent with heat-balance error data obtained in supercritical CO_2 . However, the heat-balance error data for $m < 0.045$ kg/s show the opposite trend, i.e., steeper slope than that shown in Figure D6b. Mass-flow rates lower than 0.045 kg/s were calculated using:

$$m = \frac{POW}{H_{out} - H_{in}}. \quad (D10)$$

In general, flow-rate measurement uncertainty within the range of $m = 0.045 - 0.24$ kg/s is given by:

$$\frac{\Delta m}{m} = \sqrt{\left(\frac{\Delta C_1}{C_1}\right)^2 + \left(\frac{0.5 \Delta \rho}{\rho}\right)^2 + \left(\frac{0.5 \Delta(\Delta p)}{\Delta p}\right)^2}. \quad (D11)$$

The estimated uncertainty in the constant C_1 is $\pm 0.08\%$ as a result of the minor effect of Reynolds number on the constant within the investigated range (White 1994).

Temperature, pressure (see Figure 10.5) and NIST software (2002) were used for the CO_2 density calculation. At pressures up to 30 MPa and temperatures up to 249.9°C (523 K), the estimated uncertainty in density (NIST 2002) varies up to 0.05% . Also, additional uncertainty in density arises from variations in density within the measured temperature uncertainty of $\pm 1.1^\circ\text{C}$. This additional uncertainty is about $\pm 1.1\%$ at $p = 8.36$ MPa and $t = 19^\circ\text{C}$, and $\pm 5.0\%$ at $p = 8.8$ MPa and $t = 35^\circ\text{C}$. Therefore, the total uncertainty in density is

$$\frac{\Delta \rho}{\rho} = \sqrt{\left(\frac{0.05}{100}\right)^2 + \left(\frac{1.1}{100}\right)^2} = 0.011 \text{ at } p = 8.36 \text{ MPa and } t = 19^\circ\text{C} \quad (D12)$$

and

$$\frac{\Delta \rho}{\rho} = \sqrt{\left(\frac{0.05}{100}\right)^2 + \left(\frac{5.0}{100}\right)^2} = 0.05 \text{ at } p = 8.8 \text{ MPa and } t = 35^\circ\text{C}. \quad (\text{D13})$$

However, the vast majority of the experimental data were obtained at pressure of 8.36 MPa. Therefore, the uncertainty value of 0.011 was used below.

Pressure-drop measurement uncertainties for PDT-FT-1/1 are according to Section D.3.

Hence,

- for $m_{min} = 46 \text{ g/s}$ $\Delta m = \pm 5.7 \text{ g/s}$ (or $\pm 12.5\%$) at $t = 19^\circ\text{C}$ and $p = 8.36 \text{ MPa}$, and
- for $m_{max} = 155 \text{ g/s}$ $\Delta m = \pm 2.4 \text{ g/s}$ (or $\pm 1.6\%$) at $t = 19^\circ\text{C}$ and $p = 8.36 \text{ MPa}$.

D.5 Mass Flux

Mass flux, G , is based on mass-flow rate measurements. The uncertainty, ΔG , includes an error in the estimation of the cross-sectional flow area, $A_{fl} = 5.1 \cdot 10^{-5} \text{ m}^2$. The test section is a tube of 8.058 mm ID and 10 mm OD, made of Inconel 600, with tolerances of $\pm 0.02 \text{ mm}$. The uncertainties are as follows:

For ID $\Delta D = \pm 0.02 \text{ mm}$ (or $\pm 0.25\%$),

For OD $\Delta D_{ext} = \pm 0.02 \text{ mm}$ (or $\pm 0.20\%$), and

For A_{flow} $\Delta A_{fl} = \frac{\pi D \Delta D}{2} = \pm 0.253 \text{ mm}^2$ (or $\pm 0.50\%$).

The uncertainty, ΔG , is obtained from the following equation:

$$\frac{\Delta G}{G} = \sqrt{\left(\frac{\Delta m}{A_{fl} G}\right)^2 + \left(\frac{m \Delta A_{fl}}{A_{fl}^2 G}\right)^2} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta A_{fl}}{A_{fl}}\right)^2}. \quad (\text{D14})$$

For the range of interest, the uncertainties, ΔG , are

for $G_{min} = 902 \text{ kg/m}^2 \cdot \text{s}$ ($m_{min} = 46 \text{ g/s}$) $\Delta G = \pm 112.8 \text{ kg/m}^2 \cdot \text{s}$ (or $\pm 12.5\%$), and

for $G_{max} = 3039 \text{ kg/m}^2 \cdot \text{s}$ ($m_{max} = 155 \text{ g/s}$) $\Delta G = \pm 49.8 \text{ kg/m}^2 \cdot \text{s}$ (or $\pm 1.6\%$).

D.6 Electrical Resistivity

Electrical resistivity is a calculated value (for details, see Equation (C2)) that is based on measured values of electrical resistance, heated length and tube diameters.

The accuracy of the micro-ohmmeter used in test-section electrical resistance measurements is $\pm 0.04\%$ of the reading (its readings are in milliohms). The uncertainties in ID and OD are $\Delta D = \Delta D_{ext} = \pm 0.02 \text{ mm}$, and in L it is $\Delta L = \pm 0.5 \text{ mm}$.

For a given electrical resistivity, the uncertainty $\Delta \rho$ is given by

$$\frac{\Delta \rho_{el}}{\rho_{el}} = \sqrt{\left(\frac{0.04}{100}\right)^2 + \left(\frac{\Delta D_{ext}}{D_{ext}}\right)^2 + \left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta L}{L}\right)^2}. \quad (\text{D15})$$

The uncertainty in $\Delta \rho_{el}$ ($\rho_{el} = 104.3 \cdot 10^{-8} \text{ Ohm} \cdot \text{m}$) is

- for $L = 2461 \text{ mm}$ $\Delta \rho_{el} = \pm 0.212 \cdot 10^{-8} \text{ Ohm} \cdot \text{m}$ (or $\pm 0.20\%$).

D.7 Total Test-Section Power

The total test-section power is obtained by measuring the current through a 2000 A/100 mV current shunt and the voltage across the test section. These signals are fed into a power-measuring unit (PMU), where the test-section voltage is scaled down to a 1-V level. Both the voltage and current signals are fed into isolation amplifiers and then into instrumentation amplifiers with outputs of 0 – 10 V. The amplifier outputs are fed to the computer analog inputs and represent a full-scale voltage of 175 V and a full-scale current of 2000 A. These signals are multiplied in the computer program to represent a 0 – 350 kW power level.

Calibration of the power measurement unit was performed *in situ*. Test-section voltage and current inputs were removed from the PMU. Simulated inputs were used to check the calibration of the unit. A comparison between the computer readings and the calibrated simulated inputs was used to create a curve fit for the DAS to correct for the differences. The voltage input from 0 – 110 V DC was simulated with a DC power supply and verified with a multimeter. The current shunt input was simulated with a calibrator for inputs from 10 to 100 mV, which represents 200 – 2000-A range:

Accuracy of current shunt	±0.25% of reading
Error due to current shunt resistance change	±0.02%
A/D accuracy	±0.025% of f.s., 10.00 V

The uncertainty, ΔPOW_{TS} , in power measurements (the power is a product of U and I) is given by

$$\frac{\Delta POW_{TS}}{POW_{TS}} = \sqrt{\left(\frac{0.25}{100}\right)^2 + \left(\frac{0.02}{100}\right)^2 + \left(\frac{0.1}{100}\right)^2 + \left(\frac{0.09}{100}\right)^2 + \left(\frac{\Delta I_1}{I}\right)^2 + \left(\frac{\Delta U_1}{U}\right)^2}. \quad (D16)$$

The first term is the accuracy of the current shunt, the second term is the effect of a temperature change on the current shunt, the third term is the error in the test-section voltage drop from the PMU output of $\Delta U = +0.1\%$ (0.10 V) up to 100 V, the fourth term is the error in the test-section current from the PMU output with a maximum offset of $\Delta I = +0.09\%$ (0.75 A) at 800 A, and the fifth and sixth terms are the $\pm 0.025\%$ uncertainties introduced by the AC/DC conversion process for reading the current ($\Delta I_1 = \pm 0.5$ A) and ($\Delta U_1 = \pm 0.04$ V) for reading the voltage, respectively.

For the power range, POW_{TS} , from 3.0 to 35.0 kW, and for $L = 2.208$ m, the corresponding values of voltage drop and current are

- $POW_{TS \min} = 3000$ W $U = 16.0$ V, $I = 188$ A, and
- $POW_{TS \max} = 35,000$ W $U = 54.6$ V, $I = 641$ A.

The uncertainty in ΔPOW_{TS} is as follows:

- For $POW_{TS \min} = 3000$ W $\Delta POW_{TS} = \pm 13.9$ W (or $\pm 0.46\%$), and

- For $POW_{TS\ max} = 35,000\ \text{W}$ $\Delta POW_{TS} = \pm 106.4\ \text{W}$ (or $\pm 0.30\%$).

D.8 Average Heat Flux

The uncertainty in heat flux, Δq_{ave} , involves the uncertainties in the total test-section power (see Section D.7) and in the heated area measurements, ΔA_h , where $A_h = \pi D L$. The uncertainty in ID is $\Delta D = \pm 0.02\ \text{mm}$, and in L it is $\Delta L = \pm 0.5\ \text{mm}$. Thus, ΔA_h can be calculated from

$$\frac{\Delta A_h}{A_h} = \sqrt{\left(\frac{\Delta D}{D}\right)^2 + \left(\frac{\Delta L}{L}\right)^2}. \quad (\text{D17})$$

The uncertainty in A_h ($A_h = 55,895.4\ \text{mm}^2$) is

- for $L = 2208\ \text{mm}$ and $D = 8.058\ \text{mm}$ $\Delta A_h = \pm 78.3\ \text{mm}^2$ (or $\pm 0.14\%$).

Then, the uncertainty in q_{ave} can be computed from

$$\frac{\Delta q_{ave}}{q_{ave}} = \sqrt{\left(\frac{\Delta POW_{TS}}{POW_{TS}}\right)^2 + \left(\frac{\Delta A_h}{A_h}\right)^2}, \quad (\text{D18})$$

which, for the given power values, results in

- $q_{ave\ min} = 53.7\ \text{kW}$ ($POW_{TS} = 3.0\ \text{kW}$) $\Delta q = \pm 0.28\ \text{kW/m}^2$ (or $\pm 0.53\%$), and
- $q_{ave\ max} = 626.2\ \text{kW}$ ($POW_{TS} = 35.0\ \text{kW}$) $\Delta q = \pm 2.46\ \text{kW/m}^2$ (or $\pm 0.39\%$).

However, Equation (D18) does not account for the uncertainties related to the heat loss, which are subtracted from the applied heat flux (for details, see Section 10.3.8), because the heat loss was negligible, i.e., less than 0.5%.

D9 Uncertainties in Heat-Transfer Coefficient

Local HTC is as follows:

$$HTC = \frac{q}{t_w^{\text{int}} - t_b}. \quad (\text{D19})$$

Uncertainty in the temperature difference is

$$\frac{\Delta(t_w^{\text{int}} - t_b)}{t_w^{\text{int}} - t_b} = \pm \frac{\sqrt{(\Delta t_w^{\text{int}})^2 + (\Delta t_b)^2}}{t_w^{\text{int}} - t_b}, \quad (\text{D20})$$

where uncertainty in t_w^{int} is taken as uncertainty in t_w^{ext} and uncertainty in t_b is taken as uncertainty in t_{out} .

And uncertainty ΔHTC is:

$$\frac{\Delta HTC}{HTC} = \sqrt{\left(\frac{\Delta q}{q}\right)^2 + \left(\frac{\Delta(t_w^{\text{int}} - t_b)}{t_w^{\text{int}} - t_b}\right)^2}. \quad (\text{D21})$$

D.10 Uncertainties in Thermophysical Properties near Pseudocritical Point

Uncertainties in thermophysical properties (NIST 2002) near the pseudocritical point within the uncertainty range of the measured value of bulk-fluid temperature ($\Delta t = \pm 0.4^\circ\text{C}$) are as follows (for example, at $p = 8.38 \text{ MPa}$ ($t_{pc} = 36.7^\circ\text{C}$)):

$$\Delta\rho = \pm 7\%; \Delta H = \pm 2.5\%; \Delta c_p = 4.5\%; \Delta k = \pm 2\%, \text{ and } \Delta\mu = \pm 7\%.$$

D.11 Heat-Loss Tests

Heat loss is an important component of the total heat-balance analysis. Heat loss from the test section, HL_{TS} , to the surrounding area was measured at various wall temperatures, with electrical power applied to the test section (the loop was previously evacuated to minimise heat removal through the coolant). This test provided (i) an indication of the difference between the measured external wall temperatures and ambient temperature, and (ii) data (voltage and current applied to the test section) to calculate the heat loss from the test section.

To perform the heat loss power test, a small power supply was used.

The temperature difference between the external wall temperatures and ambient temperature at zero power was found to be $\pm 0.2^\circ\text{C}$ (i.e., within the accuracy range for the thermocouples); with an increase in power to the test section, the difference ($\Delta t = t_w^{ave} - t_{amb}$) increases. This temperature difference permits the evaluation of the heat loss from the test section to the surrounding area as follows:

$$HL_{TS} = POW_{TS} = f(\Delta t), \quad (\text{D22})$$

or, as calculated,

$$HL_{TS} = POW_{TS} = U I, \quad (\text{D23})$$

where U is the voltage drop over the test section, and I is the current through the test-section wall. This heat loss test, compared to the usual zero-power test, eliminates uncertainties that are related to the estimation of the thermophysical properties of CO_2 . This test also eliminates flow-measurement uncertainties and uncertainties that are incurred when measuring very small temperature differences ($0.5 - 1^\circ\text{C}$) between the inlet and outlet bulk-fluid temperatures.

The heat-loss power test was performed with the insulated reference test section (heated length of 2.208 m). The heat loss assessed from these tests, as a function of the wall-ambient temperature difference, ($t_w^{ave} - t_{amb}$), is shown in Figure D7, and can be approximated by the following equation:

$$HL_{TS} = 0.47 (t_w^{ave} - t_{amb}) \text{ [W]}. \quad (\text{D24})$$

There were some non-uniformities in the temperature distribution along the heated length. These non-uniformities were caused by the power clamps and structural support elements for the test section, which acted as heat sinks. Therefore, a conservative approach (maximum possible heat loss and therefore, minimum HTC value) was taken, i.e., only two external wall thermocouples (TEC01 and TEC24), which are located in the same cross-sections

as TEC02 and TEC23, respectively, but 180° apart, were not taken into account (see Figure 10.6).

For local heated lengths, the following formula would apply:

$$HL_{TS} \Big|_{L_\ell} = HL_{TS} \Big|_{L=2.208m} \frac{L_\ell}{2.208} [kW], \quad (D25)$$

where L_ℓ is in metres.

In general, heat loss was negligible, i.e., less than 0.5%.

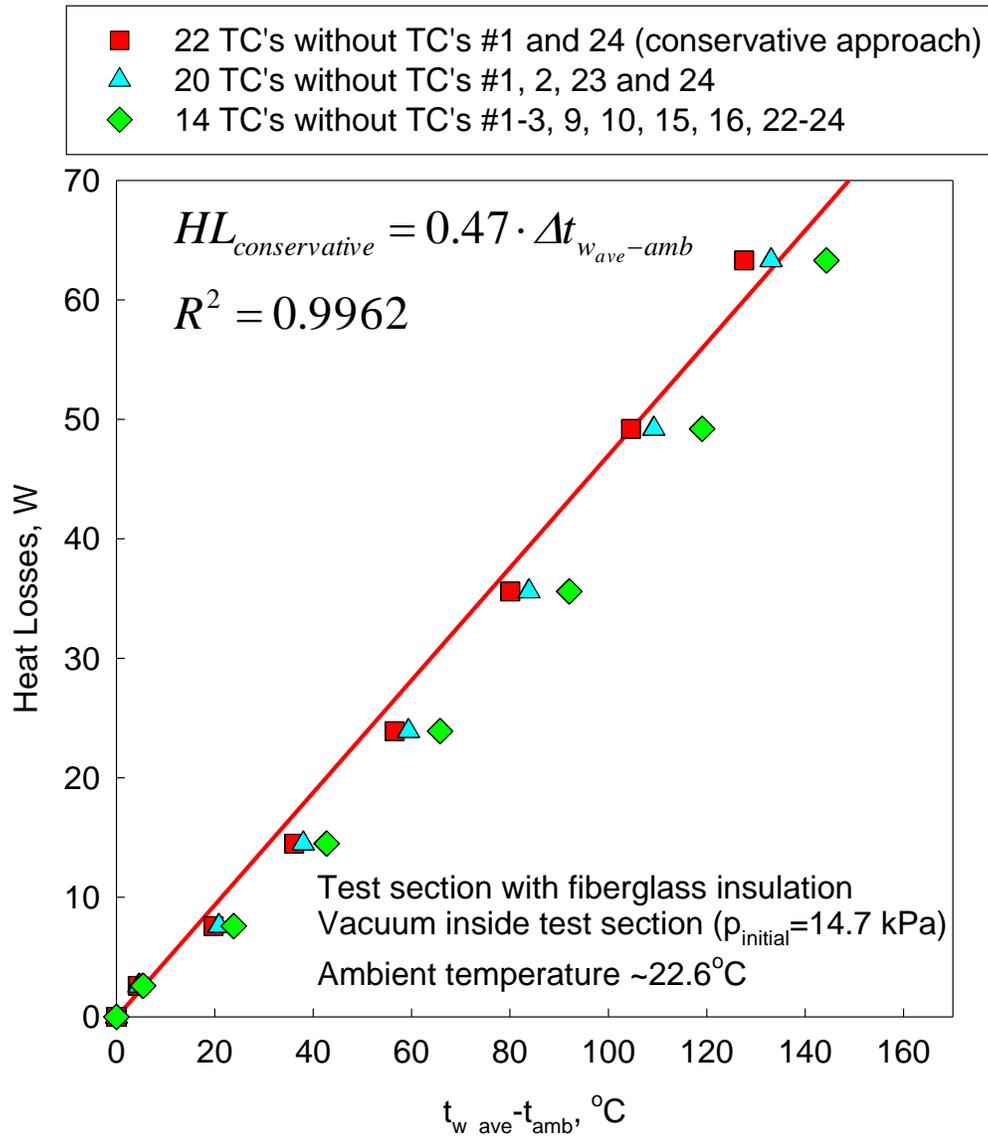


Figure D7. Heat loss from test section: Direct electrical heating of test section, heated length of 2.208 m, and loop vacuumed.

D.12 Heat-Balance Evaluation near Pseudocritical Region

For each run, an error in the heat balance was calculated using the following expression:

$$\Delta_{HB} = \frac{POW - HL - m(H_{out} - H_{in})}{POW} \cdot 100\% . \quad (D26)$$

In general, an analysis of errors in the heat-balance data shows that, at mass-flux values equal to or higher than $900 \text{ kg/m}^2\text{s}$, at medium and high values of power ($POW \geq 5 \text{ kW}$) and at the inlet and outlet bulk-fluid temperatures below or beyond the pseudocritical region (i.e., t_{in} and $t_{out} < t_{pc} - 2^\circ\text{C}$ or t_{in} and $t_{out} > t_{pc} + 2^\circ\text{C}$), these errors are within $\pm 4\%$.

Increased values of heat-balance error (i.e., more than $\pm 4\%$) at lower values of power ($POW < 5 \text{ kW}$) and at inlet or outlet bulk-fluid temperatures within the pseudocritical region (i.e., $t_{pc} - 2^\circ\text{C} < t_{in} < t_{pc} + 2^\circ\text{C}$ or $t_{pc} - 2^\circ\text{C} < t_{out} < t_{pc} + 2^\circ\text{C}$) can be explained with the following (see Table D4 and Figure D8).

At lower values of power, the increase in bulk-fluid enthalpy is relatively small. However, uncertainties in bulk-fluid enthalpy within the pseudocritical region are larger for the same uncertainty range in bulk-fluid temperature, compared to the enthalpy values' uncertainties that correspond to temperatures far from the pseudocritical region.

Table D4. Maximum uncertainties in ΔH calculations near pseudocritical point ($p_{out}=8.36 \text{ MPa}$, $t_{pc}=36.7^\circ\text{C}$, $t_{in}=21^\circ\text{C}$, $m=0.1 \text{ kg/s}$, and $G=2000 \text{ kg/m}^2\text{s}$).

t_b	H_b	Uncertainty in H_b at $\Delta t_b=+0.4^\circ\text{C}$	Uncertainty in H_b at $\Delta t_b=-0.4^\circ\text{C}$	$\Delta H_b=H_{out}-H_{in}$	Max uncertainty in ΔH_b
$^\circ\text{C}$	kJ/kg	kJ/kg	kJ/kg	kJ/kg	$\%$
21	248.94	1.18	-1.19	-	-
35	313.72	4.29	-5.04	64.78	14.4
37	349.26	8.51	-7.82	100.32	16.3
41	395.75	2.56	-2.4	144.41	3.4

Also, an additional error in the heat balance appears at mass-flux values below $900 \text{ kg/m}^2\text{s}$ (see Figure D6), where the flow-measuring curve is steep. Therefore, lower values of mass flux should be measured with a smaller diameter orifice flowmeter⁸ or other type

⁸ However, orifice-plate flowmeters with a diameter of the orifice less than 12.5 mm is considered a non-standard type.

flowmeters.

Figure D8 shows an example of the heat-balance evaluation near the pseudocritical region. This graph shows that, at lower power values ($POW < 5$ kW) and at the outlet bulk-fluid temperature within the pseudocritical region, variations in bulk-fluid enthalpy difference can be up to 11.5% within the nominal uncertainty range for bulk-fluid temperatures (i.e., $\pm 0.4^\circ\text{C}$).

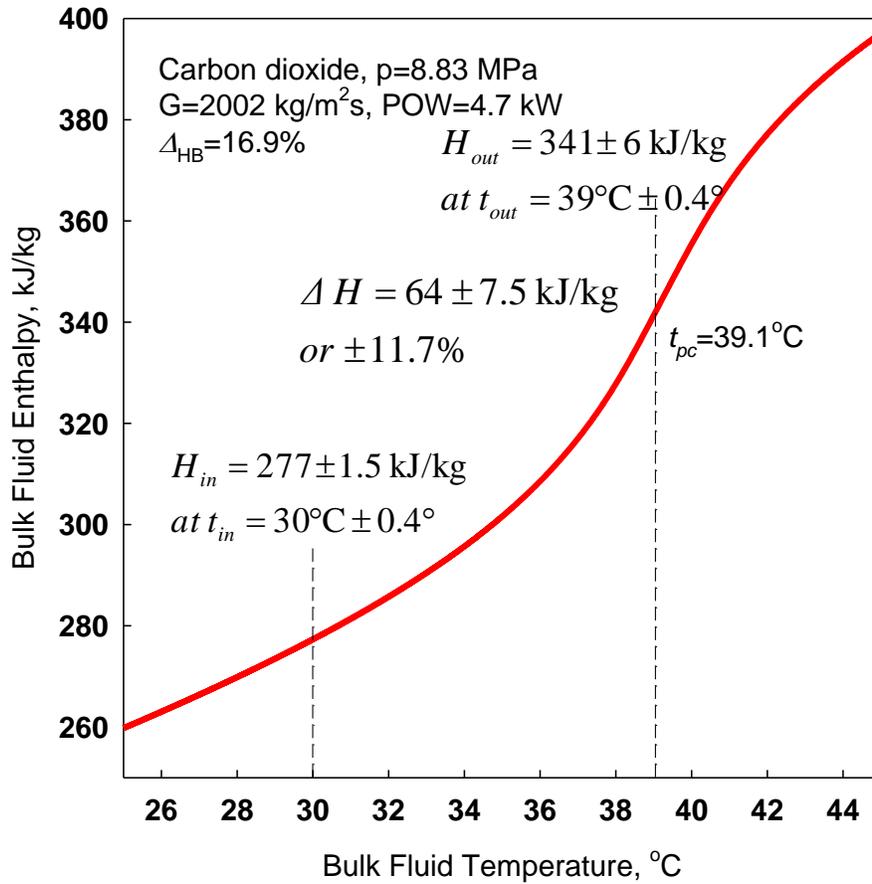


Figure D8. Heat-balance evaluation near pseudocritical region.

SYMBOLS AND ABBREVIATIONS

A	area, m ²
A_{fl}	flow area, m ²
c_p	specific heat at constant pressure, J/kg K
\bar{c}_p	averaged specific heat within the range of $(T_w - T_b)$; $\left(\frac{H_w - H_b}{T_w - T_b} \right)$, J/kg K
D	inside diameter, m
D_{ext}	external diameter, m
D_{hy}	hydraulic diameter, m; $\left(\frac{4 A_{fl}}{P_{wetted}} \right)$
f	friction factor; $\left(\frac{\sigma_w}{G^2} \right)$ $\left(\frac{8 \rho}{\sigma_w} \right)$
f_d	drag coefficient
G	mass flux, kg/m ² s; $\left(\frac{m}{A_{fl}} \right)$
g	gravitational acceleration, m/s ²
H	specific enthalpy, J/kg
h	heat transfer coefficient, W/m ² K
HL	heat loss, W
I	current, A
k	thermal conductivity, W/m K
L	heated length, m
L_{tot}	total length, m
m	mass-flow rate, kg/s; (ρV)
p	pressure, MPa
POW	power, W
Q	heat-transfer rate, W

q	heat flux, W/m^2 ; $\left(\frac{Q}{A_h}\right)$
q_v	volumetric heat flux, W/m^3 ; $\left(\frac{Q}{V_h}\right)$
R	molar gas constant, 8.31451 J/mol K
R_a	arithmetic average surface roughness, μm
R_{bend}	radius of bending (for tube)
R_{el}	electrical resistance, Ohm
r	radial coordinate or radius, m; regression coefficient
T	temperature, K
t	temperature, $^{\circ}\text{C}$
U	voltage, V
u	axial velocity, m/s
V	volume, m^3 or volumetric flow rate, m^3/s
V_m	molar volume, m^3/mol
v	radial velocity, m/s
x	axial coordinate, m
y	radial distance; $(r_0 - r)$, m
z	axial coordinate, m

Greek Letters

α	thermal diffusivity, m^2/s ; $\left(\frac{k}{c_p \rho}\right)$
β	volumetric thermal expansion coefficient, 1/K
Δ	difference
Δ_{HB}	error in heat balance, %
δ	thickness, mm
ε	dissipation of turbulent energy
μ	dynamic viscosity, Pa s
π	reduced pressure; $\left(\frac{P}{P_{cr}}\right)$

P	perimeter, m
ρ	density, kg/m ³
ρ_{el}	electrical resistivity, Ohm·m
σ	dispersion
σ_w	viscous stress, N/m ²
ν	kinematic viscosity, m ² /s
ζ	friction coefficient

Non-dimensional Numbers

Ga Galileo number; $\left(\frac{g D^3}{\nu^2} \right)$

Gr Grashof number; $\left(\frac{g \beta \Delta T D^3}{\nu^2} \right)$

Gr_q modified Grashof number; $\left(\frac{g \beta q_w D^4}{k \nu^2} \right)$

Nu Nusselt number; $\left(\frac{h D}{k} \right)$

Pr Prandtl number; $\left(\frac{\mu c_p}{k} \right) = \left(\frac{\nu}{\alpha} \right)$

$\overline{\text{Pr}}$ averaged Prandtl number within the range of $(t_w - t_b)$; $\left(\frac{\mu \bar{c}_p}{k} \right)$

Re Reynolds number; $\left(\frac{G D}{\mu} \right)$

Ra Raleigh number; **(Gr Pr)**

St Stanton number; $\left(\frac{\text{Nu}}{\text{Re Pr}} \right)$

Symbols with an overline at the top denote average or mean values (e.g., $\overline{\text{Nu}}$ denotes average (mean) Nusselt number).

Subscripts or superscripts

ac acceleration

amb ambient

ave	average
b	bulk
cal	calculated
cr	critical
cr sect	cross section
dht	deteriorated heat transfer
el	electrical
ext	external
f	fluid
fl	flow
fm	flowmeter
fr	friction
g	gravitational
h	heated
HB	Heat Balance
hor	horizontal
hy	hydraulic
in	inlet
int	internal
iso	isothermal
ℓ	liquid or local
m	molar
max	maximum
meas	measured
min	minimum
nom	nominal or normal
0	constant properties, scale, reference, characteristic, initial, or axial value
out	outlet or outside
OD	outside diameter
pc	pseudocritical
T	value of turbulent flow

TS	test section
th	threshold value
tot	total
v	volumetric
vert	vertical
w	wall

Abbreviations and acronyms widely used in the text and list of references

AC	Alternating Current
A/D	Analog-to-Digital (conversion)
A/I	Analog Input
AECL	Atomic Energy of Canada Limited (Canada)
AERE	Atomic Energy Research Establishment (UK)
AGR	Advanced Gas-cooled Reactor
AIAA	American Institute of Aeronautics and Astronautics
AIChE	American Institute of Chemical Engineers
ANS	American Nuclear Society
ASME	American Society of Mechanical Engineers
ASHRAE	American Society of Heating, Refrigerating and Air-conditioning Engineers
AWG	American Wire Gauge
BWR	Boiling Water Reactor
CANDU	CANada Deuterium Uranium nuclear reactor
CFD	Computational Fluid Dynamics
CHF	Critical Heat Flux
CRL	Chalk River Laboratories, AECL (Canada)
DAS	Data Acquisition System
DC	Direct Current
DOE	Department Of Energy (USA)
DP	Differential Pressure
emf	electromagnetic force
ENS	European Nuclear Society
EU	European Union

EXT	EXternal
FA	Fuel Assembly
FBR	Fast Breeder Reactor
FM	FlowMeter
F/M	Ferritic-Martensitic (steel)
FR	Fuel Rod
f.s.	full scale
FT	Flow Transducer
GIF	Generation IV International Forum
HMT	Heat Mass Transfer
HT	Heat Transfer
HTC	Heat Transfer Coefficient
HTD	Heat Transfer Division
HTR	High Temperature Reactor
HVAC & R	Heating Ventilating Air-Conditioning and Refrigerating
IAEA	International Atomic Energy Agency (Vienna, Austria)
ID	Inside Diameter
INEEL	Idaho National Engineering and Environmental Laboratory (USA)
IP	Intermediate-Pressure turbine
IPPE	Institute of Physics and Power Engineering (Obninsk, Russia)
JAERI	Japan Atomic Energy Research Institute
JSME	Japan Society of Mechanical Engineers
KAERI	Korea Atomic Energy Research Institute (South Korea)
KPI	Kiev Polytechnic Institute (nowadays National Technical University of Ukraine “KPI”) (Kiev, Ukraine)
KP-SKD	Channel Reactor of Supercritical Pressure (in Russian abbreviations)
LP	Low-Pressure turbine
LOCA	Loss Of Coolant Accident
LOECC	Loss Of Emergency Core Cooling
Ltd.	Limited
LWR	Light Water Reactor

MEI	Moscow Power Institute (Moscow, Russia) (In Russian abbreviations)
MIT	Massachusetts Institute of Technology (Cambridge, MA, USA)
MOX	Mixed Oxide (nuclear fuel)
NASA	National Aeronautics and Space Administration (USA)
NIST	National Institute of Standards and Technology (USA)
NPP	Nuclear Power Plant
OD	Outside Diameter
PC	Personal Computer
PDT	Pressure Differential Transducer
PH.D.	Philosophy Degree
PLC	Programmable Logic Controller
ppb	parts per billion
ppm	parts per million
PT	Pressure Tube or Pressure Transducer
PWAC	Pratt & Whitney AirCraft
PWR	Pressurized Water Reactor
R	Refrigerant
RAS	Russian Academy of Sciences
RBMK	Reactor of Large Capacity Channel type (in Russian abbreviations)
RDIPe	Research and Development Institute of Power Engineering (Moscow, Russia) (NIKIET in Russian abbreviations)
R&D	Research and Development
RMS	Root-Mean-Square (error or surface roughness)
RPV	Reactor Pressure Vessel
RSC	Russian Scientific Centre
RT	propulsion fuel (in Russian abbreviations)
RTD	Resistance Temperature Detector
SCP	SuperCritical Pressure
SCR	SuperCritical Reactor
SCW	SuperCritical Water
SCWO	SuperCritical Water Oxidation technology

SCWR	SuperCritical Water-cooled Reactor
SFL	Supercritical Fluid Leaching
SFR	Sodium Fast Reactor
SKD	SuperCritical Pressure (in Russian abbreviations)
SMR	Steam-Methane-Reforming process
SS	Stainless Steel
T	fuel (in Russian abbreviation)
TC	ThermoCouple
TE	TEmperature
TECO	TEmperature of CO ₂
TS	Test Section
TsKTI	Central Boiler-Turbine Institute (St.-Petersburg, Russia) (in Russian abbreviations)
UCG	Uranium-Carbide Grit pored over with calcium (nuclear fuel)
UK	United Kingdom
U.K.A.E.A.	United Kingdom Atomic Energy Authority (UK)
UNESCO	United Nations Educational, Scientific and Cultural Organization (Paris, France)
US or USA	United States of America
USSR	Union of Soviet Socialist Republics
VHTR	Very High-Temperature Reactor
VNIAM	All-Union Scientific-Research Institute of Atomic Machine Building (Russia) (in Russian abbreviations)
VTI	All-Union Heat Engineering Institute (Moscow, Russia) (in Russian abbreviations)
wt	weight
WWPR	Water-Water Power Reactor (“VVER” in Russian abbreviations)

REFERENCES

- Bezrodny, M.K., Pioro, I.L. and Kostyuk, T.O., 2005. Transfer Processes in Two-Phase Thermosyphon Systems. Theory and Practice, (In Russian), 2nd edition, Augmented and Revised, Fact Publ. House, Kiev, Ukraine, 704 pages.
- Coleman, H.W. and Steele, W.G., 1999. Experimentation and Uncertainty Analysis for Engineers, 2nd edition, J. Wiley & Sons, Inc., New York, USA, 275 pages.
- Gortyshov, Yu.F., Dresvyannikov, F.N., Idiatullin, N.S. et al., 1985. Theory and Technique of Thermophysical Experiment, Textbook for Technical Universities, (In Russian), Editor V.K. Shchukin, Energoatomizdat Publishing House, Moscow, Russia, 360 pages.
- Guide to the Expression of Uncertainty in Measurement, 1995. Corrected and Reprinted, International Bureau of Weights and Measures and other International Organizations, 101 pages.
- Guo, Y., Bullock, D.E., Pioro, I.L. and Martin, J., 2006. Measurements of sheath temperature profiles in LVRF bundles under post-dryout heat transfer conditions in Freon, Proceedings of the 14th International Conference on Nuclear Engineering (ICONE-14), July 17-20, Miami, Florida, USA, Paper #89621, 9 pages.
- Hardy, J.E., Hylton, J.O., McKnight, T.E. et al., 1999. Flow Measurement Methods and Applications. J. Wiley & Sons, Inc., New York, NY, USA, pp. 207–208.
- Holman, J.P., 1994. Experimental Methods for Engineers, 6th edition, McGraw-Hill, Inc., New York, NY, USA, 616 pages.
- Kichigin, A.M. and Pioro, I.L., 1992. Analysis of methods for detecting dryout and burnout fluxes, Heat Transfer Research, 24 (7), pp. 957–964.

Kirillov, P.L., Pomet'ko, R.S., Smirnov et al., 2005. Experimental study on heat transfer to supercritical water flowing in vertical tubes, Proceedings of the International Conference GLOBAL-2005 "Nuclear Energy Systems for Future Generation and Global Sustainability, Tsukuba, Japan, October 9–13, Paper No. 518, 8 pages.

Leung, L.K.H., Piro, I.L. and Bullock, D.E., 2003. Post-dryout surface-temperature distributions in a vertical freon-cooled 37-element bundle, Proceedings of the 10th International Topical Meeting on Nuclear Reactor Thermal Hydraulics (NURETH-10), Seoul, Korea, October 5-9, Paper C00201, 13 pages.

Mark's Standard Handbook for Mechanical Engineers, 1996. Editors: Eu.A. Avallone and Th. Baumeister III, McGraw-Hill, New York, NY, USA, p. 16-20.

Moffat, R.J., 1988. Describing the uncertainties in experimental results, Experimental Thermal and Fluid Science, 1, pp. 3–17.

NIST Reference Fluid Thermodynamic and Transport Properties—REFPROP, 2002. NIST Standard Reference Database 23 (on CD: Executable with Source plus Supplemental Fluids in ZIP File), Version 7.0, E.W. Lemmon, M.O. McLinden and M.L. Huber, National Institute of Standards and Technology, Boulder, CO, U.S. Department of Commerce, August.

Piro, I.L., 1982. Investigation of closed two-phase thermosiphons used for cooling of melting converters, (In Russian), Applied Problems of Heat Transfer and Hydrodynamics, Naukova Dumka, Publishing House, Kiev, Ukraine, pp. 76–79.

Piro, I.L., 1992. Maximum heat-transferring capacity of two-phase thermosiphons with separate vapor and condensate streams, Heat Transfer Research, 24 (4), pp. 535–542.

Piro I.L., 1999. Experimental evaluation of constants for the Rohsenow pool boiling

correlation, *International Journal of Heat and Mass Transfer*, 42, pp. 2003–2013.

Pioro, I.L. and Cheng, S.C., 1998. Concise literature survey of the Russian and selected western publications devoted to the heat transfer and hydraulic resistance of a fluid at near critical and supercritical pressures, Report UO-MCG-TH-98003, prepared for AECL CRL, Mechanical Engineering Department, University of Ottawa, November, 148 pages.

Pioro, I.L., Cheng, S.C., Vasić, A. and Felisari, R., 2000. Some problems for bundle CHF prediction based on CHF measurements in simple flow geometries, *Nuclear Engineering and Design*, 201 (2–3), pp. 189–207.

Pioro, I.L., Groeneveld, D.C., Leung, L.K.H. et al., 2002a. Comparison of CHF measurements in horizontal and vertical tubes cooled with R-134a, *International Journal of Heat and Mass Transfer*, 45 (22), pp. 4435–4450.

Pioro, I.L., Groeneveld, D.C., Doerffer, S.S. et al., 2002b. Effects of flow obstacles on the critical heat flux in a vertical tube cooled with upward flow of R-134a, *International Journal of Heat and Mass Transfer*, 45 (22), pp. 4417–4433.

Pioro, I.L., Groeneveld, D., Cheng, S.C. et al., 2001. Comparison of CHF measurements in R-134a cooled tubes and the water CHF look-up table, *International Journal of Heat and Mass Transfer*, 44 (1), pp. 73–88.

Pioro, I.L. and Kalashnikov, A.Yu., 1988. Maximum heat transfer in two-phase thermosyphons with external down-flow channel, *Applied Thermal Sciences*, 1 (2), pp. 62–67.

Pioro, I.L. and Khartabil, H.F., 2005. Experimental study on heat transfer to supercritical carbon dioxide flowing upward in a vertical tube, *Proceedings of the 13th International Conference on Nuclear Engineering (ICONE-13)*, Beijing, China, May 16–20, Paper No. 50118, 9 pages.

Pioro, L.S. and Pioro, I.L., 1997. Industrial Two-Phase Thermosyphons, Begell House, New York, NY, USA, 288 pages.

Pis'mennyy, E.N., Razumovskiy, V.G., Maevskiy E.M. et al., 2005. Experimental study on temperature regimes to supercritical water flowing in vertical tubes at low mass fluxes, Proceedings of the International Conference GLOBAL-2005 "Nuclear Energy Systems for Future Generation and Global Sustainability, Tsukuba, Japan, October 9–13, Paper No. 519, 9 pages.

The Flow and Level Handbook, 2000. OMEGA Engineering, Inc., 21st Century Edition, Vol. MM, (Transactions, Vol. 4, p. 22).

The Temperature Handbook, 2000. OMEGA Engineering, Inc., 21st Century Edition, Vol. MM.

Topping, J. 1971. Errors of Observation and Their Treatment, 3rd edition, Chapman and Hall Ltd., London, UK, 119 pages.